# HAND WRITTEN NOTES OF COMPLETE SYLLABUS OF ELECTRICITY, MAGNETISM AND EMT FOR BSc 1st YEAR.

# CHAPTER-I Vector Analysis



•**Scalar quantity:—** A scalar quantity which is completely specified by its magnitude or size is called Scalar quantity.

# VECTORS

Ex:— Mass, length, time, Volume, Density, temp., speed etc.

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Vectors: It is the quantity and it has present in both magnitude and direction are called Vector.

Magnitude ⇒ Magnitude refers to the size of an object or its speed while travelling.

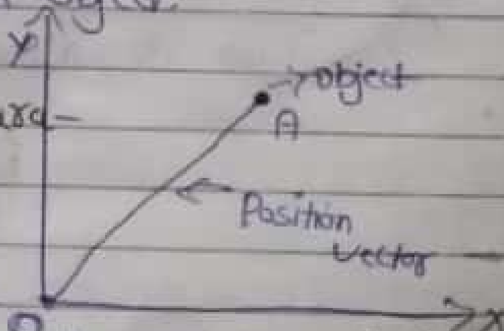
Direction ⇒ Direction means that a vector simply moves a one point to another point is called direction.

Ex. of Vector: Acceleration, displacement, velocity, Force, Momentum, Electric field etc.

• Vector physical quantity cannot be added, substracted by using simple Algebra. But, they added subtracted, multiplying by the using the Vector algebra.

Position Vector: The position vector is a vector representing the position of an object.

⇒ It consists of two characteristics:

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i) It gives the straight line distance of the object from the origin. § $\vec{OA}$ = (position vector)

ii) It gives the direct$^n$ of the object with r.t origin.

Object: A material thing that can be seen or touched is called object.

Types of Vectors :— Vectors are divided into two categories,

(i) Polar vectors :— A vector quantity whose direction is along the direction of the motion of the body of the particle is known as Polar vector.

Ex:— Displacement, velocity, momentum, force etc.

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⇒ **Negative Vector:** A negative vector is a vector which points in the direction opposite to the reference positive direction is called Negative vector.

⇒ **Equal Vector:** Two vectors are said to be Equal Vectors if they have Same magnitude and ... Called Equal Vector.

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⇒ **Co-initial Vector:** Two vectors (have) are said to be Co initial vector If, they have Common initial point, is called Co-initial vector.

(Co-Same) Initial → Starting point

⇒ **Co-linear vector:** Two vectors having Equal or Un-~~Equal~~ equal magnitude but Either act on Same line, parallel lines and directions, parallel lines are in opposite direction are called Co-linear Vector.

Resultant vector $(\vec{R}) = \vec{P} + \vec{Q}$

⇒ **Unit vector:** A vector which have only one magnitude is called unit vector.

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$ unit vector,

• Unit vector is denoted by (Caps) ^.

⇒ **Zero-vector:** A vector having zero magnitude are known as Zero-Vector. And, it is also known as null vector.

It is representing by $\vec{0}$.

(ii) Axial vector :— A vector quantity whose direction is along the axis of rotation of body or particle is called axial ~~vector~~.

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Ex :- Angular velocity $(\vec{\omega})$, Angular acceleration $(\vec{\alpha})$, torque $(\vec{\tau})$ etc

3 Resolution of vector: The process of splitting up a vector into two or more vector is known as Resolution of vectors

## Addition of vectors :—

### (i) Triangle law of vector addition :—



If two vectors are represented both in magnitude and direction by two sides of triangle taken in the same order, then the resultant of these vectors is represented both in magnitude and direction by third side of the triangle taken in the opposite order.

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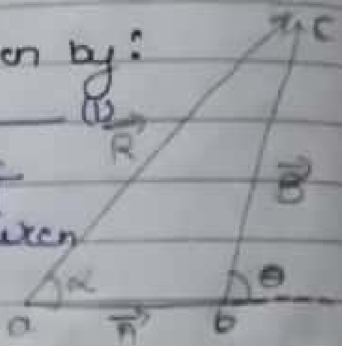
Let two vectors $\vec{A}$ and $\vec{B}$ are represented by two sides ab and bc of a triangle abc. then the resultant of these vectors $\vec{A}$ and $\vec{B}$ is $\vec{R}$, which is represented by the third side ac of the triangle abc.




i.e $\vec{R} = \vec{A} + \vec{B}$

• Magnitude of Resultant vector $\vec{R}$ is given by :

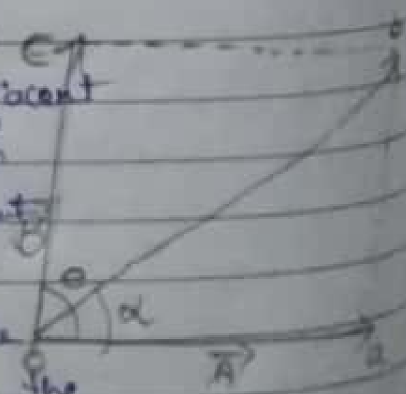
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad \text{—— (1)}$$

where A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$. $\theta$ is the angle between vectors $\vec{A}$ and $\vec{B}$.

Definition of the resultant vector $\vec{R}$ with vector $\vec{A}$ is given by :

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}} \quad \text{—— (2)}$$

### (ii) Parallelogram law of Vectors addition :—




If two vectors are represented both in magnitude and direction by the two adjacent sides of these ~~two vectors, then the~~ llgm drawn from a point, then the resultant vector of these vectors is represented both in magnitude and direction by the diagonal of the llgm passing through the same point.

Let vectors $\vec{A}$ and $\vec{B}$ be represented by the two adjacent sides of oa and oc of llgm oabc, then the resultant vector $\vec{R}$ of these vectors is given by :—

$$\vec{R} = \vec{A} + \vec{B}$$ , Magnitude of resultant vector $\vec{R}$ is given by :—

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$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad \text{——} \quad (1)$$

Where A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$. $\theta$ is the angle betn ~~vectors~~ $\vec{A}$ and $\vec{B}$.

- ~~Resultant of~~ Directn of the Resultant vector $\vec{R}$ with ~~velocity~~ vector $\vec{A}$ is:

$$\boxed{\Rightarrow \tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$

## • Properties of vector addition :—

(i) Vectors obeys commutative law :

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

(ii) Vectors addn obeys associative law:

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C}).$$

## * Scalar and dot product of two vectors :—

The scalar and dot product of two non-zero vectors $\vec{A}$ and $\vec{B}$ is :—

$\vec{A} \cdot \vec{B} = AB\cos\theta$, where $\vec{A}$ is the mag. of vector $\vec{A}$ and B is the mag. of $\vec{B}$ and $\theta$ is the angle betn A and B.

$\Rightarrow$ The scalar or dot product of two vectors gives the scalar physical quantity.

- **Vector or cross product of two vectors :—**

The vector or cross product of two vectors of non-zero vector $\vec{A}$ and $\vec{B}$ is

$$\vec{A} \times \vec{B} = (AB\sin\theta)\hat{n}$$

where A, B are the magnitudes of $\vec{A}$ and $\vec{B}$. So, $\theta$ is the angle betn the vector $\vec{A}$ and $\vec{B}$. i.e $\hat{n}$ is the unit vector ⊥lar to the plane containing vectors $\vec{A}$ and $\vec{B}$.



## *Definition of Field :—

If a physical quantity (scalar or vector) varies from point to point in space. It can be expressed as a continuous function of the position of a point in the region or space. Such a continuous funct. is called function of position. The region in which function is specifies the physical quantity is called Field.

### ⇒ Types of Fields:

(i) Scalar Field :— The region of space in which scalar quantity has unique value at every point is called scalar field.

S.F can be represented as:—

$$\phi = \phi(\vec{x})$$

$$\phi = \phi(x, y, z).,\ \phi \text{ is a scalar quantity.}$$

Example:— Distribution of temp., Electric potential, density etc.

(ii) Vector Field:— The region of space in which vector quantity has unique value at every point is called Vector field.

V.F is repre. as:—

$$\vec{A} = \vec{A}(\vec{x}).$$

$$\vec{A} = \vec{A}(x, y, z).$$

Example:— Electric field intensity, magnetic field intensity, gravitational field etc.

# • DEL OPERATOR:

The del operator ($\vec{\nabla}$) is called the vector differential operator. This operator was introduced by Sir Hamilton and read as del.

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$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}.$$

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## • Laplacian operator:

$\vec{\nabla}.\vec{\nabla} = \nabla^2$ is called the Laplacian operator.

$$\vec{\nabla}.\vec{\nabla} = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right].\left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right].$$

$\hat{i}.\hat{i}=1$
$\hat{j}.\hat{j}=1$
$\hat{k}.\hat{k}=1$

$$\Rightarrow \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

## • Gradient of Scalar Field: ($\phi$)

When a scalar field ($\phi$) is operated upon by a del operator ($\vec{\nabla}$), we get a vector function which is called the gradient of scalar field ($\phi$).

• Gradient of Scalar field:

$$\text{grad.}\phi = \vec{\nabla}\phi = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right]\phi.$$

$$= \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k},$$

Gradient of $\phi$ is a vector quantity.

Ex. of V. quantities which are expressed as the gradient of scalar fields are:—

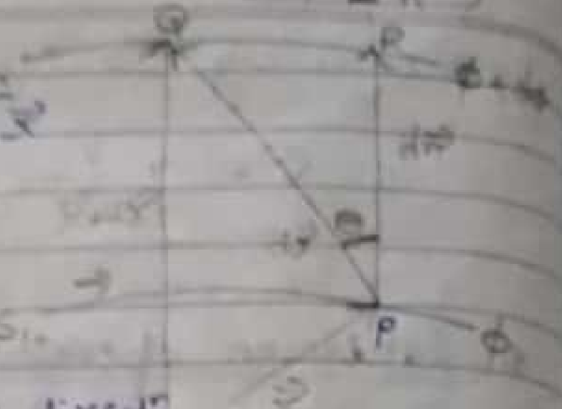
(i) Gravitational force $\vec{F} = -\vec{\nabla}U$ { U is Gravitational potential Energy

(ii) Electric field due to static charge distribution $\vec{E} = -\vec{\nabla}V$ i.e. V is Electric Potential.

# ⇒ Physical meaning of Gradient of the scalar function:



Let two level surfaces $S_1$ and $S_2$ represents the scalar function $\phi$ and $\phi + d\phi$.



Let P and Q are the two points on these surfaces such that $\vec{OP} = \vec{x}$ and $\vec{OQ} = \vec{x} + d\vec{x}$.

Such that $\vec{PQ} = d\vec{x}$.

Let the least dist$^n$ bet$^n$ the two surfaces $S_1$ and $S_2$ in $|\vec{PR}|$ in the direct$^n$ of normal at point P. $\hat{n}$ be the unit vector along $\vec{PR}$ and $\vec{PR} = d\vec{n}$.

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$$dn = dx\cos\theta = \hat{n} . d\vec{x} \quad \text{—— (1)}$$

$\theta$ is the angle bet$^n$ $\hat{n}$ and $d\vec{x}$

The rate of increase of $\phi$ at P in the direct$^n$ of PQ will be $\frac{d\phi}{dx}$ and it becomes maximum if $dx$ is minimum.

minim$^m$ value of $dx$ is along $\vec{PR}$.

Max$^m$ rate of $\phi = \frac{d\phi}{dn}$ along $\hat{n}$.

$$d\phi = \frac{d\phi}{dn} dn.$$

Using Eqn. (1) we get.

$$d\phi = \frac{d\phi}{dn} \hat{n} . d\vec{x} \quad \text{—— (2)}$$

$$d\phi = \vec{\nabla}\phi . d\vec{x} \quad \text{—— (3)}$$

Comparing Eqn.s (2) and (3), we get

$$\Rightarrow \vec{\nabla}\phi . d\vec{x} = \frac{d\phi}{dn} \hat{n} . d\vec{x}$$

$\vec{\nabla}\phi \cdot d\vec{r} = \frac{\partial \phi}{\partial n} \hat{n} \cdot d\vec{r}$

$\Rightarrow \vec{\nabla}\phi = \frac{d\phi}{dn} \hat{n}$ , Gradient of Scalar function $\phi$ is a vector whose magnitude at any point of is equal to the max^m rate of increase of $\phi$ and is directed along the direct^n in which the rate of change of the function $\phi$ is maximum.

**Geometrical Interpretation of Grad $\phi$ (show that $\vec{\nabla}\phi$ is ⊥ar to the level surface).**

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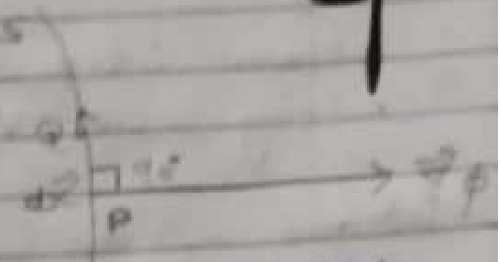



Scalar function $\phi$ is represented by a level surface. Consider one such surface S having $\phi$ = const.

Let we move along the surface S through a dist^n $dr$ from a point P to point Q. The change in $\phi$ in going from P to Q is zero.

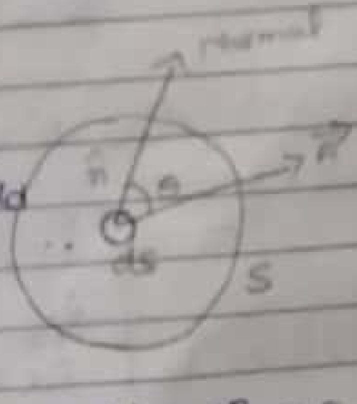
$$d\phi = 0 \quad \text{——} (1)$$

$$d\phi = \vec{\nabla}\phi \cdot d\vec{r}$$

Using Eqn (i) we get $\quad \vec{\nabla}\phi \cdot d\vec{r} = 0 \quad \text{——} (2)$

This shows that $\vec{\nabla}\phi$ is ⊥ar to $d\vec{r}$.

## Surface Integral:




Consider a surface S in the vector field $\vec{A}$. Let ds be the area of small element on this surface.

Let $\hat{n}$ be the unit vector showing the direct^n of area vector $d\vec{s}$ and $\theta$ be the angle bet^n $\vec{A}$ and $\hat{n}$.

The flux of $\vec{A}$ through the element of area ds,

$$d\phi = \vec{A} \cdot d\vec{s} = \vec{A} \cdot ds\hat{n}$$

The total Flux through the whole surface S is given by, $\phi = \oiint \vec{A}\cdot d\vec{s}$.



$\oiint \vec{A}\cdot d\vec{s}$ is called surface integral of $\vec{A}$ through the whole surface S

Now, $\phi = \oiint \vec{A}\cdot d\vec{s}$.

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$$= \oiint (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(ds_x\hat{i} + ds_y\hat{j} + ds_z\hat{k})$$

$$= \oiint (A_x ds_x + A_y ds_y + A_z ds_z) \text{ Ans}$$

Volume integral:— If we consider a surface enclosing volume V, then $\iiint_V \vec{A}\,dv$ is the volume integral of the vector field $\vec{A}$ for the whole volume V over the surface.

*Curl of a Vector:—

The cross product of $\vec{\nabla}$ and a vector field $\vec{A}$ is called the curl of vector field $\vec{A}$ and is written by $\vec{\nabla}\times\vec{A}$.

Let a vector field $\vec{A}$ is represented as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\text{Curl } \vec{A} = \vec{\nabla}\times\vec{A} = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right]\times(A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix} = \hat{i}\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ A_y & A_z \end{vmatrix} - \hat{j}\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ A_x & A_z \end{vmatrix}$$

$$+ \hat{k}\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ A_x & A_y \end{vmatrix} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{i} - \left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right]$$

$$+ \hat{k}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right].$$

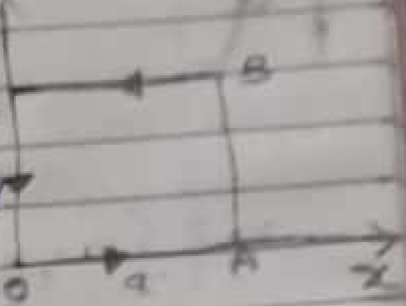
Then x, y and z Components of Curl $\vec{A}$ are given by:—

$\Rightarrow |(\vec{\nabla}\times\vec{A})_x| = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]$

$\Rightarrow |(\vec{\nabla}\times\vec{A})_y| = \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)$ $\Rightarrow |(\vec{\nabla}\times\vec{A})_z| = \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$

i.e. Curl of vector field ~~which~~ is a vector quantity.

Example 45:— Verify stoke's theorem for the function $\vec{E} = x(\hat{i}x + \hat{j}y)$, integrated around the square in the plane $z=0$, whose sides are along the lines $x=0,\ y=0,\ x=a,\ y=a$.




Solution:— OABC is a square with sides $= a$

$$\oint_C \vec{E}\cdot d\vec{l} = \int_{OA}\vec{E}\cdot d\vec{l} + \int_{AB}\vec{E}\cdot d\vec{l} + \int_{BC}\vec{E}\cdot d\vec{l} + \int_{CO}\vec{E}\cdot d\vec{l}$$

$$\therefore \int_{OA}\vec{E}\cdot d\vec{l} = \int_0^a x(x\hat{i} + y)\cdot\hat{i}\,dx = \int_0^a x^2 dx$$

$$= \frac{a^3}{3} \quad \{\text{along } OA,\ y=0\}$$

$\left\{\frac{x^{2+1}}{2+1} = \frac{x^3}{3} = \frac{a^3}{3}\right\}$

$$\int_{AB}\vec{E}\cdot d\vec{l} = \int_0^a x(\hat{i}x + \hat{j}y)\cdot\hat{j}\,dy = \int_0^a xy\,dy = \frac{a^3}{3} \quad \{\text{along } BC,\ x=a\}$$

$$\int_{BC}\vec{E}\cdot d\vec{l} = \int_0^a x(\hat{i}x + \hat{j}y)(-\hat{i})dx = -\int_0^a x^2 dx = -\frac{a^3}{3} \quad \{\text{along } BC,\ y=a\}$$

$$\int_{CO}\vec{E}\cdot d\vec{l} = \int_0^a x\{\hat{i}x + y(-\hat{j}]\cdot\hat{j}\,dy = 0 \quad \{\text{along } CO,\ x=0\}$$

$$= \oint_C \vec{E}\cdot d\vec{l} = \tfrac{1}{3}a^3 + \frac{a^3}{2} - \frac{a^3}{3} + 0 = \frac{a^3}{2}.$$

Acc. to Stoke's theorem:—

$\oint_C \vec{F}\cdot d\vec{r} = \iint_S \text{curl}\,\vec{F}\cdot d\vec{S}$

$\Rightarrow \text{curl}\,\vec{F} = \vec{\nabla}\times\vec{F}$

$= \vec{\nabla}\times x(x\hat{i}+y\hat{j})$

$\iint_S \text{curl}\,\vec{F}\cdot d\vec{S} = \iint_S \vec{\nabla}\times x(x\hat{i}+y\hat{j})\cdot d\vec{S} = \iint_S \left[\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right]\times(x^2\hat{i}+xy\hat{j})\cdot d\vec{S}$

Now, $\vec{\nabla}\times x(x\hat{i}+y\hat{j}) = \left[\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right]\times(x^2\hat{i}+xy\hat{j})$

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$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & xy & 0 \end{vmatrix} = \hat{i}\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ xy & 0\end{vmatrix} - \hat{j}\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2 & 0\end{vmatrix} + \hat{k}\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2 & xy\end{vmatrix}$

$= \hat{i}\left[0-\frac{\partial}{\partial z}(xy)\right] + \hat{j}\left[0-\frac{\partial}{\partial z}x^2\right] + \hat{k}\left\{\frac{\partial}{\partial x}(xy)-\frac{\partial}{\partial y}(x^2)\right\}$

$= 0+0+y\hat{k} = y\hat{k}. \Rightarrow \iint_S \text{curl}\,\vec{F}\cdot d\vec{S} = \iint_S y\hat{k}\cdot dS\hat{k}$

$= \iint y\,dS = \int_0^a\int_0^a y\,dx\,dy = a\int_0^a y\,dy = a\times\frac{a^2}{2} = \frac{a^3}{2}$

Hence, Stoke's theorem is verified. Ans

not change with ...

(iii) Unit of resistivity is:— $\Omega m$ , unit conductivity $\Omega^{-1} m^{-1}$.

# ★ Gauss's Divergence Theorem:—

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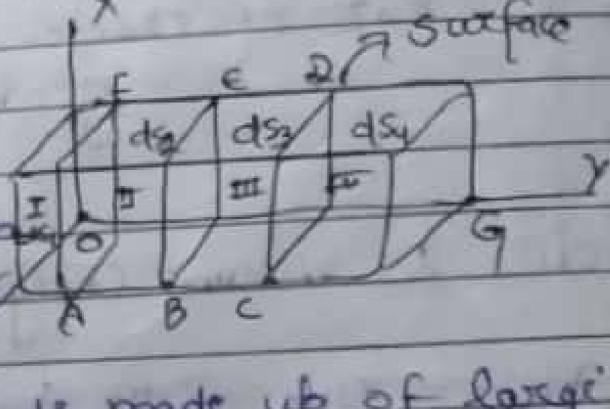
$$\oiint_S \vec{A}\cdot d\vec{s} = \iiint_V (\vec{\nabla}\cdot\vec{A})\, d\vec{v}$$




Proof:— Let us consider the surface S in the region. Consider $\vec{A}$ be the vector field in the given region. Let the vol$^m$ is made up of large no. of elementary volumes in the form of parallelopiped

Consider the $J^{th}$ parallelopiped over the vol$^m$. V surface $d\vec{s_j}$.

Now, Consider the whole volume is diveded into three Elementary volume I, II, III. $\oiint_{S_j} \vec{A} \, d\vec{s_j}$ is the normal directed outward from the vol$^m$ under consideration. The Elementary vol$^m$ I is directed in inward of Elementary vol$^m$ II, Elementary vol$^m$ II is inward the Elementary vol$^m$ III and so on. The sum of integral of Elementary vol$^m$ will be cancel each other.

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Now,

$$\Sigma \oiint_{S_j} \vec{A} \, d\vec{s_j} = \oiint_{S} \vec{A} . d\vec{s} \Rightarrow \oiint_{S} \vec{A} \, d\vec{s} = \Sigma \iint_{S_j} \vec{A} \, d\vec{s_j} \quad \text{--- (1)}$$

$\Rightarrow$ Multiply and divide $\nabla v_j$ on Right hand side,

$$\oiint_{S} \vec{A} . d\vec{s} = \Sigma . \frac{1}{\nabla v_j} \left[ \iint_{S_j} \vec{A} \, d\vec{s_j} \right] \nabla v_j \quad \text{--- (2)}$$

Now, let us suppose the vol$^m$. of the surface S divided in elementary vol$^m$. we get.

$$\oiint_{S} \vec{A} \, d\vec{s} = \lim_{\nabla v_j \to 0} \frac{1}{\nabla v_j} \left[ \iint_{S_j} \vec{A} \, d\vec{s_j} \right] \nabla v_j \quad \text{--- (3)}$$

$$\lim_{\nabla v_j \to 0} \frac{1}{\nabla v_j} \left[ \iint_{S_j} \vec{A} \, d\vec{s_j} \right] \cancel{\nabla v_j} = (\vec{\nabla} . \vec{A}) \quad \text{--- (4)}$$

Put (3) Eqn. in $(\vec{\nabla} \vec{A})$.

$$\oiint_S \vec{A}\cdot d\vec{s} = \sum (\vec{\nabla}\cdot\vec{A})\cdot\Delta V_i$$

Now, $\Delta V_i \to 0$ i.e. $\sum \Delta V_i$ becomes volm integral of over volm (V).

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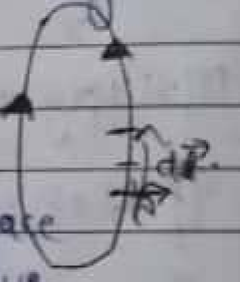
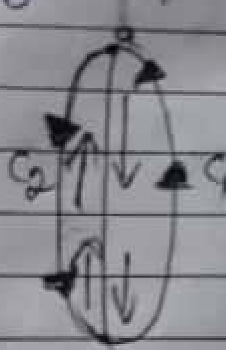
So, $$\boxed{\oiint_S \vec{A}\cdot d\vec{s} = \iiint_V (\vec{\nabla}\cdot\vec{A})\,dv.}$$ which is the Gauss's divergence theorem.

# Stoke's Theorem.

The line integral of vector field $\vec{A}$ around any closed curve is equal to the surface integral of the curl of vector field $\vec{A}$ taken over any surface S which the curve is bounding edge.

$$\oint_C \vec{A}\cdot d\vec{l} = \iint_S (\vec{\nabla}\times\vec{A})\cdot d\vec{s}$$




Proof:— Let S be the surface of the edge, C be the curve. $\vec{A}$ be the vector field acting on the surface enclosed by the closed surface.

$\vec{A}$ along a closed curve is,

$$\oint_C \vec{A}\cdot d\vec{l}.$$

Now, where, $d\vec{l}$ be the small element of the area path.

If we divide the area enclosed by the closed surface $C_1$ and $C_2$ with two equal parts drawn a line a and b. We have a two equal pa of closed parts curve $C_1$ and $C_2$.

Now, $$\oint_C \vec{A}\cdot d\vec{l} = \oint_{C_1} \vec{A}\cdot d\vec{l} + \oint_{C_2} \vec{A}\cdot d\vec{l} \quad \text{--- (1)}$$

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If, the area enclosed by closed curve C is divided into large no. of small elements $ds_1, ds_2$ --- $ds_n$ bounded by the curves $C_1, C_2$ --- $C_n$.

Now, $$\oint_C \vec{A}\cdot d\vec{l} = \sum \oint_{C_n} \vec{A}\cdot d\vec{l} \quad \text{--- (2)}$$

Acc. to definition of curl.

$$\Rightarrow \sum \oint_{C_n} \vec{A}\cdot d\vec{l} = \text{Curl}\,\vec{A}\cdot d\vec{s_n} \quad \text{--- (3)}.$$

$\Rightarrow$ put $\oint_{C_n} \vec{A}\cdot d\vec{l}$ in Eqn. (2), we get,

$$\oint_C \vec{A}\cdot d\vec{l} = \sum \text{Curl}\,\vec{A}\cdot d\vec{s_n} \quad \text{--- (4)}.$$

$\Rightarrow ds_n$ is the surface area of the curve.

Now, $ds_n \to 0$, $\sum \text{Curl}\,\vec{A}\, d\vec{s_n} = \int_S \text{Curl}\,\vec{A}\cdot d\vec{s}$

In Eqn. (4) we get

$$\Rightarrow \oint_C \vec{A}\cdot d\vec{l} = \iint_S \text{Curl}\,\vec{A}\, d\vec{s}.$$

$$\Rightarrow \oint_C \vec{A}\cdot d\vec{l} = \iint_S (\vec{\nabla}\times\vec{A})\, d\vec{s}.$$

i.e which is the Stoke's theorem. Ans.

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• Prove that $\text{div}.(\text{curl}\ \vec{A}) = 0$, where $\vec{A}$ is a vector function.

Solution:— $\text{div}\ (\text{curl}\ \vec{A}) = 0$

Acc. to Stoke's theorem.

$$\oint_C \vec{A}.d\vec{l} = \iint_S (\vec{\nabla} \times \vec{A})\, d\vec{s}.$$

Due to curl of $\vec{A}$, the closed curve reduces to a point

$$\oint_C \vec{A}.d\vec{l} = 0.$$

Then, $$\iint_S (\vec{\nabla} \times \vec{A})\, d\vec{s} = 0 \quad \text{—— (1)}$$

Acc. to Gauss div. Theorem,

$$\iint_S \vec{A}.d\vec{s} = \iiint_V (\vec{\nabla}.\vec{A})\, dv \quad \text{—— (2)}$$

Taking curl on both sides, we get.

$$\Rightarrow \iint_S \text{curl}.\vec{A}.d\vec{s} = \iiint_V \text{curl}(\vec{\nabla}.\vec{A})\, dv. \quad \text{—— (3)}$$

$$\text{or} \quad \iint_S (\vec{\nabla} \times \vec{A})\, d\vec{s} = \iiint_V \vec{\nabla} \times (\vec{\nabla}.\vec{A})\, dv \quad \text{—— (4)}$$

$$\Rightarrow \iiint_V \vec{\nabla}.(\vec{\nabla} \times \vec{A})\, dv.$$

Using eqn (1),

$$\Rightarrow \iiint_V \vec{\nabla}.(\vec{\nabla} \times \vec{A})\, dv = 0$$

$$\Rightarrow \vec{\nabla}.(\vec{\nabla} \times \vec{A}) = 0 \quad \text{or} \quad \text{div}.(\text{curl}\ \vec{A}) = 0$$

- Prove that curl (grad $\phi$) = 0, where $\phi$ is a scalar function.

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$\Rightarrow$ Acc to stoke's theorem,

$$\oint_C \vec{A}\, d\vec{l} = \iint_S (\vec{\nabla} \times \vec{A}) d\vec{s}.$$

$\Rightarrow \vec{A} = \vec{\nabla}\phi.$

$$\oint_C \vec{\nabla}\phi \cdot d\vec{l} = \iint_S (\vec{\nabla} \times \vec{\nabla}\phi) d\vec{s}$$

$$\Rightarrow \oint_C \vec{\nabla}\phi\, d\vec{l} = 0. \text{ i.e } \iint_S (\vec{\nabla} \times \vec{\nabla}\phi)\, d\vec{s} = 0$$

$\vec{\nabla} \times \vec{\nabla}\phi = 0$ or curl (grad. $\phi$) = 0.

# ★ GREEN'S THEOREM

The Green's theorem is the extension of stoke's theorem and Gauss's divergence theorem.

If, $\phi$ and $\psi$ are the scalar function.

$$\Rightarrow \iiint_V (\psi \nabla^2 \phi - \phi \nabla^2 \psi) dv = \iint_S \psi \vec{\nabla}\phi - \phi \vec{\nabla}\psi$$

Proof :- Let a vector field $\vec{A} = \psi \vec{\nabla}\phi - \phi \vec{\nabla}\psi$

Acc to Gauss divergence theorem $\iint_S \vec{A} \cdot d\vec{s} = \iiint (\vec{\nabla} \cdot \vec{A}) dv$

$$\iint_S (\psi \vec{\nabla}\phi - \phi \vec{\nabla}\psi) \cdot d\vec{s} = \iiint_V \vec{\nabla} (\psi \vec{\nabla}\phi - \phi \vec{\nabla}\psi) dv$$

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$$\vec{\nabla}.(\psi\vec{\nabla}\phi - \phi\vec{\nabla}\psi) = \vec{\nabla}(\psi.\vec{\nabla}\phi) - \vec{\nabla}(\phi\vec{\nabla}\psi).$$

$$= \psi\vec{\nabla}.\vec{\nabla}\phi + \vec{\nabla}\phi.\vec{\nabla}\psi - (\phi\vec{\nabla}.\vec{\nabla}\psi + \vec{\nabla}\phi.\vec{\nabla}\psi.$$

$$= \psi\nabla^2\phi \cancel{+\vec{\nabla}\phi.\vec{\nabla}\psi} - \phi\nabla^2\psi \cancel{-\vec{\nabla}\phi.\vec{\nabla}\psi}$$

$$= \psi\nabla^2\phi - \phi\nabla^2\psi.$$

$$\Rightarrow \iint_S (\psi\vec{\nabla}\phi - \phi\vec{\nabla}\psi).d\vec{s} = \iiint_V \vec{\nabla}.(\psi\vec{\nabla}\phi - \phi\vec{\nabla}\psi)\,dv$$

$$\boxed{\Rightarrow \text{or} \quad \iiint_V \vec{\nabla}.(\psi\vec{\nabla}\phi - \phi\vec{\nabla}\psi)\,dv = \iint_S (\psi\vec{\nabla}\phi - \phi\vec{\nabla}\psi).d\vec{s}.}$$

Which is Green's theorem. Ans

Ex. 6:-

Solution:- $\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}.$

$|\vec{x}| = (x^2+y^2+z^2)^{1/2}.$

$$\frac{1}{|\vec{x}|} = \frac{1}{(x^2+y^2+z^2)^{1/2}} \Rightarrow \frac{1}{|\vec{x}|} = (x^2+y^2+z^2)^{-1/2}.$$

$$\Rightarrow \text{grad}\left[\frac{1}{|\vec{x}|}\right] = \vec{\nabla}\left[\frac{1}{|\vec{x}|}\right]$$

$$= \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right].\left[x^2+y^2+z^2\right]^{-1/2}.$$

$$= \hat{i}\frac{\partial}{\partial x}\left[x^2+y^2+z^2\right]^{-1/2} + \hat{j}\frac{\partial}{\partial y}\left[x^2+y^2+z^2\right]^{-1/2} + \hat{k}\frac{\partial}{\partial z}\left[x^2+y^2+z^2\right]^{-1/2}.$$

$$= \hat{i}\,\frac{\partial}{\partial x}\left[-\tfrac{1}{2}\right]\left[x^2+y^2+z^2\right]^{-3/2}(2x) + \hat{j}\left[-\tfrac{1}{2}\right]\left[x^2+y^2+z^2\right]^{-3/2}(2y)$$

$$+ \hat{k}\left[-\tfrac{1}{2}\right]\left[x^2+y^2+z^2\right]^{-3/2}(2z).$$

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$$= \frac{-x\hat{i}}{(x^2+y^2+z^2)^{3/2}} - \frac{y\hat{j}}{(x^2+y^2+z^2)^{3/2}} - \frac{z\hat{k}}{(x^2+y^2+z^2)^{3/2}}$$

$$\text{grad}\left[\frac{1}{|\vec{x}|}\right] = -\frac{(x\hat{i}+y\hat{j}+z\hat{k})}{(x^2+y^2+z^2)^{3/2}} = -\frac{\vec{x}}{|\vec{x}|^3} = -\frac{\hat{x}}{|x|^2}$$

$$[\text{i.e. } \vec{x} = |\vec{x}|\,\hat{x}].$$

① ⇒ Irrotational field.

Given $\vec{E} = 6xy\,\hat{i} + (3x^2-3y^2)\hat{j}$

$\Rightarrow \nabla \times \vec{E} = 0$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 6xy & (3x^2-3y^2) & 0 \end{vmatrix}$$

$$\Rightarrow \hat{i}\left[\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(3x^2-3y^2)\right] + \hat{j}\left[\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial z}(6xy)\right]$$

$$+ \hat{k}\left[\frac{\partial}{\partial x}(3x^2-3y^2) - \frac{\partial}{\partial z}6xy\right] = 0$$

$$\Rightarrow \hat{i}[0] + \hat{j}[0] + \hat{k}[6x-6x] = 0$$

$$\Rightarrow 0\hat{i} + 0\hat{j} + [6x-6x]\hat{k} = 0$$

i.e. $\vec{\nabla} \times \vec{E} = 0$, so $\vec{E}$ is irrotational field.

$= \frac{\vec{x}}{|\vec{x}|^3} = -\frac{\vec{x}}{|\vec{x}|^2}$ proved $\left[\text{i.e. } \vec{x} = \frac{|\vec{x}|}{x}\right]$

$\hat{x} = \frac{\vec{x}}{|\vec{x}|}$

$|\vec{x}| = x$

(2) $\vec{\nabla}\left[\frac{1}{x^3}\right] = \frac{-3\vec{x}}{x^5}$

$\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$

$x = (x^2+y^2+z^2)^{1/2}$

and $x^3 = (x^2+y^2+z^2)^{3/2}$

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Now, $\left[\frac{1}{x^3}\right] = \frac{1}{(x^2+y^2+z^2)^{3/2}}$

$= (x^2+y^2+z^2)^{-3/2}$

$\vec{\nabla}\left[\frac{1}{x^3}\right] = \vec{\nabla}(x^2+y^2+z^2)^{-3/2}$

$= \left[\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right]\left[x^2+y^2+z^2\right]^{-3/2}$

$-\frac{3}{2}-1 = -\frac{5}{2}$

$= -\hat{i}\left(\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}(2x) - \hat{j}\left(\frac{3}{2}\right)\left[x^2+y^2+z^2\right]^{-5/2}(2y)$

$- \hat{k}\left(\frac{3}{2}\right)\left[x^2+y^2+z^2\right]^{-5/2}(2z).$

$= -\frac{3x\hat{i}}{(x^2+y^2+z^2)^{5/2}} - \frac{3y\hat{j}}{(x^2+y^2+z^2)^{5/2}} - \frac{3z\hat{k}}{(x^2+y^2+z^2)^{5/2}}$

$= -\frac{3}{x^5}(x\hat{i}+y\hat{j}+z\hat{k}) = -\frac{3}{x^5}\vec{x}.$

$\boxed{\text{i.e. } \vec{\nabla}\left[\frac{1}{x^3}\right] = -\frac{3}{x^5}\vec{x}.}$

i.e. Hence proved.

Example-8

(3) $\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$ is the p.v grad. of $x^n$.

$\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$.

$\bar{x} = |\vec{x}| = (x^2+y^2+z^2)^{1/2}$

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and $|\vec{x}|^n = (x^2+y^2+z^2)^{n/2}$.

grad. of$(x^n) = \vec{\nabla}|\vec{x}|^n$.

$$= \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right]\left[x^2+y^2+z^2\right]^{n/2}.$$

$$= \frac{n}{2}\hat{i}\left[x^2+y^2+z^2\right]^{\frac{n}{2}-1}(2x) + \frac{n}{2}\hat{j}\left[x^2+y^2+z^2\right]^{\frac{n}{2}-1}(2y) +$$

$$\frac{n}{2}\hat{k}\left[x^2+y^2+z^2\right]^{\frac{n}{2}-1}(2z).$$

$\frac{n-2}{2}$

$$= nx\hat{i}\left[x^2+y^2+z^2\right]^{\frac{n-2}{2}} + ny\hat{j}\left[x^2+y^2+z^2\right]^{\frac{n-2}{2}}$$

$$+ nz\hat{k}\left[x^2+y^2+z^2\right]^{\frac{n-2}{2}}.$$

$$= n(x^2+y^2+z^2)^{\frac{n-2}{2}}\left[x\hat{i} + y\hat{j} + z\hat{k}\right].$$

$$= n|\vec{x}|^{n-2}(x\hat{i} + y\hat{j} + z\hat{k}).$$

grad.$(x^n) = n.|\vec{x}|^{n-2}.\vec{x}$.

$$\boxed{\vec{\nabla}x^n = n.|\vec{x}|^{n-2}.\vec{x}}$$

Example-9.

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{A}$ is constt. vector,

grad $(\vec{A}\cdot\vec{r})$.

$\nabla(\vec{A}\cdot\vec{r}) = \vec{A}$.

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$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$.

$(\vec{A}\cdot\vec{r}) = xA_x + yA_y + zA_z$.

$\nabla(\vec{A}\cdot\vec{r}) = \left[\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right]\left[xA_x + yA_y + zA_z\right]$

$= A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = \vec{A}$.

Example-10.

$\vec{\nabla}(uv) = u\vec{\nabla}v + v\vec{\nabla}u$. u and v are Scalars.

$\vec{\nabla}(uv) = \left[\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right](uv)$.

$= \frac{d}{dx}(uv)\hat{i} + \frac{d}{dy}(uv)\hat{j} + \frac{d}{dz}(uv)\hat{k}$.

$= u.\frac{dv}{dx}\hat{i} + v.\frac{du}{dx}\hat{i} + u\frac{dv}{dy} + v.\frac{du}{dy}\hat{j}$

$+ u\frac{dv}{dz}\hat{k} + v\frac{du}{dz}\hat{k}$.

$= u\left[\hat{i}\frac{d}{dx}(v) + \hat{j}\frac{d(v)}{dy} + \hat{k}\frac{d}{dz}(v)\right] + v\left[\hat{i}\frac{d}{dx}(u) + \hat{j}\frac{d}{dy}(u) + \hat{k}\frac{d}{dz}(u)\right]$.

$$= u\left[\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right]v + v\left[\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right]u$$

$$\boxed{\vec{\nabla}(uv) = u\vec{\nabla}v + v\vec{\nabla}u.}$$ Ans.

24 Example-11.

$\phi(x,y,z) = 3x^2y - y^3z^2$, $\phi$ at the point $(1,-2,-1)$.

Sol $\Rightarrow$ $\phi(x,y,z) = 3x^2y - y^3z^2$.

$$\phi = \vec{\nabla}\phi = \left[\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right]\left[3x^2y-y^3z^2\right]$$

$$= \hat{i}\frac{\partial}{\partial x}(3x^2y-y^3z^2) + \hat{j}\frac{\partial}{\partial y}(3x^2y-y^3z^2) + \hat{k}\frac{\partial}{\partial z}\left[3x^2y-y^3z^2\right].$$

$$\vec{\nabla}\phi = \hat{i}(6xy) + \hat{j}(3x^2-3y^2z^2) + \hat{k}(-2y^3z)$$

$(x,y,z) = (1,-2,1)$ then $\vec{\nabla}\phi(1,-2,1) = -12\hat{i} - 9\hat{j} - 16\hat{k}$

Example-12.

Find $\vec{\nabla}\phi$ $\quad \phi = x^{3/2} + y^{3/2} + z^{3/2}$.

$$\vec{\nabla}\phi = \left[\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right]\left(x^{3/2}+y^{3/2}+z^{3/2}\right).$$

$$= \hat{i}\frac{\partial}{\partial x}\left(x^{3/2}+y^{3/2}+z^{3/2}\right) + \hat{j}\frac{\partial}{\partial x}\left(x^{3/2}+y^{3/2}+z^{3/2}\right) + \hat{k}\frac{\partial}{\partial x}\left(x^{3/2}+y^{3/2}+z^{3/2}\right).$$

$$= \frac{3}{2} x^{1/2} \hat{i} + \frac{3}{2} y^{1/2} \hat{j} + \frac{3}{2} z^{1/2} \hat{k}.$$

$$\boxed{\vec{\nabla} \phi = \frac{3}{2} \left[ x^{1/2} \hat{i} + y^{1/2} \hat{j} + z^{1/2} \hat{k} \right].}$$

25 Example-13.

$$V = x^2 y + y^2 z + z^2 x + 3xyz.$$

$$\vec{\nabla} V = \left( \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \left( x^2 y + y^2 z + z^2 x + 3xyz \right)$$

$$= \hat{i} \frac{\partial}{\partial x} \left( x^2 y + y^2 z + z^2 x + 3xyz \right) + \hat{j} \frac{\partial}{\partial y} \left( x^2 y + y^2 z + z^2 x + 3xyz \right)$$
$$+ \hat{k} \frac{\partial}{\partial z} \left[ x^2 y + y^2 z + z^2 x + 3xyz \right].$$

$$= (2xy + z^2 + 3yz) \hat{i} + (x^2 + 2yz + 3xz) \hat{j} + (y^2 + 2zx + 3xy) \hat{k}.$$

$$\vec{\nabla} V = (z^2 + 2xy + 3yz) \hat{i} + (x^2 + 2yz + 3xz) \hat{j} + (y^2 + 2zx + 3xy) \hat{k}.$$

Example -14.

Here, $V = \frac{kq}{r} = \frac{kq}{(x^2+y^2+z^2)^{1/2}}$ where, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

and $r = \sqrt{x^2+y^2+z^2}$.

$\vec{E}$ be the Electric field due to charge $q$ at distance $r$,

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$$\vec{E} = -\vec{\nabla} V = -\left[\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right]\left[\frac{Kq}{(x^2+y^2+z^2)^{1/2}}\right]$$

$$= -Kq\left[\hat{i}\left\{\frac{d}{dx}\left[\frac{1}{(x^2+y^2+z^2)^{1/2}}\right] + \frac{d}{dy}\left[\frac{1}{(x^2+y^2+z^2)^{1/2}}\right]\hat{j} + \frac{d}{dz}\left[\frac{1}{x^2+y^2+z^2}\right]\hat{k}\right]\right.$$

$$= -Kq\left[\frac{d}{dx}(x^2+y^2+z^2)^{-1/2}\hat{i} + \frac{d}{dy}(x^2+y^2+z^2)^{-1/2}\hat{j} + \frac{d}{dz}(x^2+y^2+z^2)^{-1/2}\hat{k}\right].$$

$$= -Kq\left[\left(-\tfrac{1}{2}\right)(x^2+y^2+z^2)^{-3/2}(2x) + \left(-\tfrac{1}{2}\right)(x^2+y^2+z^2)^{-3/2}(2y) + \left(-\tfrac{1}{2}\right)(x^2+y^2+z^2)^{-3/2}(2z)\right].$$

$$= -Kq\left[-\frac{x}{(x^2+y^2+z^2)^{3/2}}\hat{i} - \frac{y}{(x^2+y^2+z^2)^{3/2}}\hat{j} - \frac{z}{(x^2+y^2+z^2)^{3/2}}\hat{k}\right]$$

$$= Kq\left[\frac{(x\hat{i} + y\hat{j} + z\hat{k})}{(x^2+y^2+z^2)^{3/2}}\right].$$

$$= Kq\left[\frac{\vec{r}}{r^3}\right].$$

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Example-15-

$$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}.$$

$$\text{div } \vec{r} = \vec{\nabla}.\vec{r}$$
$$= \left[\frac{d}{dx}\hat{i}+\frac{d}{dy}\hat{j}+\frac{d}{dz}\hat{k}\right]\left[x\hat{i}+y\hat{j}+z\hat{k}\right].$$

$$= \frac{dx}{dx} + \frac{d}{dy}y + \frac{d}{dz}z = 1+1+1 = 3.$$

Example-16.

$\vec{\nabla}.(r^n\vec{r}) = (n+3)r^n$, where $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$.

$$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$$
and $r = |\vec{r}| = \sqrt{x^2+y^2+z^2}$ or $(x^2+y^2+z^2)^{1/2}$.
$$r^n = (x^2+y^2+z^2)^{n/2}$$

$$\vec{\nabla}.(r^n\vec{r}) = \left[\frac{d}{dx}\hat{i}+\frac{d}{dy}\hat{j}+\frac{d}{dz}\hat{k}\right].\left[(x^2+y^2+z^2)^{n/2}(x\hat{i}+y\hat{j}+z\hat{k}\right].$$

$$= \frac{d}{dx}(x^2+y^2+z^2)^{n/2}.x + \frac{d}{dx}(x^2+y^2+z^2)^{n/2}.y + \frac{d}{dz}(x^2+y^2+z^2)^{n/2}.z$$

product

$$= (x^2+y^2+z^2)^{n/2}.1 + x\tfrac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}(2x) + (x^2+y^2+z^2)^{n/2}.1$$
$$+ y.\tfrac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}.(2y) + (x^2+y^2+z^2)^{n/2}$$
$$1. + z.\tfrac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}(2z).$$

$= 3(x^2+y^2+z^2)^{n/2} + n(x^2+y^2+z^2)^{n/2-1}(x^2+y^2+z^2).$

$= 3(x^2+y^2+z^2)^{n/2} + n(x^2+y^2+z^2)^{n/2}.$

$= (n+3)(x^2+y^2+z^2)^{n/2} = (n+3)\,r^n$ ; Ans

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Example 18

$\vec{\nabla}\cdot\left[\frac{\vec{r}}{|\vec{r}|^3}\right] = 0$, where $\vec{r} = (x\hat{i}+y\hat{j}+z\hat{k})$

$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$r = |\vec{r}| = \sqrt{x^2+y^2+z^2}$ or $(x^2+y^2+z^2)^{1/2}$.

Now, $r^3 = (x^2+y^2+z^2)^{3/2}$.

$\vec{\nabla}\cdot\left[\frac{\vec{r}}{|\vec{r}|^3}\right] = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right]\cdot\left[\frac{x\hat{i}+y\hat{j}+z\hat{k}}{(x^2+y^2+z^2)^{3/2}}\right]$

$= \frac{\partial}{\partial x}\left[\frac{x}{(x^2+y^2+z^2)^{3/2}}\right] + \frac{\partial}{\partial y}\left[\frac{y}{(x^2+y^2+z^2)^{3/2}}\right] + \frac{\partial}{\partial z}\left[\frac{z}{(x^2+y^2+z^2)^{3/2}}\right]$

$= \frac{\partial}{\partial x}\left[x.(x^2+y^2+z^2)^{-3/2}\right] + \frac{\partial}{\partial y}\left[y.(x^2+y^2+z^2)^{-3/2}\right] +$

$\frac{\partial}{\partial z}\left[z.(x^2+y^2+z^2)^{-3/2}\right].$

$= x\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2} + y\left(-\frac{3}{2}\right)[x^2+y^2+z^2]^{-5/2}(2y) + (x^2+y^2+z^2)^{-3/2} + z\left(-\frac{3}{2}\right)[x^2+y^2+z^2]^{-5/2}\,2z.\,(x^2+y^2+z^2)^{-3/2}.$

$$= -\frac{3x^2}{(x^2+y^2+z^2)^{5/2}} + \frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3y^2}{(x^2+y^2+z^2)^{5/2}} + \frac{1}{(x^2+y^2 ...}$$

$$- \frac{3z^2}{(x^2+y^2+z^2)^{5/2}} + \frac{1}{(x^2+y^2+z^2)^{3/2}}.$$

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$$= \frac{-3(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{5/2}} + \frac{3}{(x^2+y^2+z^2)^{3/2}}$$

$$= -3(x^2+y^2+z^2)^1(x^2+y^2+z^2)^{-5/2} + \frac{3}{(x^2+y^2+z^2)^{3/2}}$$

$1 - 5/2 = \frac{2-5}{2} = -3/2$

$$= -3(x^2+y^2+z^2)^{-3/2} + \frac{3}{(x^2+y^2+z^2)^{3/2}}.$$

$$= -\frac{\cancel{3}}{(x^2+y^2+z^2)^{3/2}} + \frac{\cancel{3}}{(x^2+y^2+z^2)^{3/2}}.$$

$$= 0 \text{ Ans.}$$

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Example-19

Prove that ⇒
$\vec{\nabla}.(\vec{A}+\vec{B}) = \vec{\nabla}.\vec{A} + \vec{\nabla}.\vec{B}$.

$\vec{A}$ and $\vec{B}$ are two vectors are expressed as

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$(\vec{A}+\vec{B}) = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} + B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$(\vec{A}+\vec{B}) = (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j} + (A_zB_z)\hat{k}$.

$\vec{\nabla}.(\vec{A}+\vec{B}) = \left[\hat{i}\frac{d}{dx} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right].\left[(A_x+B_x)\hat{i} + (A_y+B_y)\hat{j} + (A_zB_z)\hat{k}\right]$.

$= \left[\frac{d}{dx}A_x + \frac{d}{dy}A_y + \frac{d}{dz}A_z\right] + \left[\frac{d}{dx}B_x + \frac{d}{dx}B_y + \frac{d}{dz}B_z\right]$.

$= \left[\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right].\left[(A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\right] + \left[\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right].\left[B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right]$.

$= \vec{\nabla}.\vec{A} + \vec{\nabla}.\vec{B}$.

<Hence proved>

Example - 20

$\vec{v} = x^2z\hat{i} - 2y^3z^2\hat{j} + xy^2z\hat{k}$.

$\vec{\nabla}.\vec{v}$ at point $(1,-1,1)$.

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Given $\vec{v} = x^2 z\hat{i} - 2y^3 z^2\hat{j} + xy^2 z\hat{k}$.

$$\vec{\nabla}\cdot\vec{v} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(x^2 z\hat{i} - 2y^3 z^2\hat{j} + xy^2 z\hat{k}\right)$$

$$= \frac{\partial}{\partial x}(x^2 z) + \frac{\partial}{\partial y}(-2y^3 z^2) + \frac{\partial}{\partial z}(xy^2 z).$$

$$\vec{\nabla}\cdot\vec{v} = 2xz - 6y^2 z^2 + xy^2.$$

Now at point $(1,-1,1)$ i.e $x=1$, $y=-1$ and $z=1$.

$$\vec{\nabla}\cdot\vec{v} = 2\times1\times1 - 6(-1)^2(1)^2 + (1)(-1)^2.$$

$$\vec{\nabla}\cdot\vec{v} = 2 - 6(1)(1) + (1).(1).$$

$$\vec{\nabla}\cdot\vec{v} = -4 + 1.$$

$$\boxed{\vec{\nabla}\cdot\vec{v} = -3} \text{ Ans.}$$

Example - 21.

$\phi = 2x^3yz^2$ {div(grad $\phi$)}.

$$\phi = \vec{\nabla}\phi = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right](2x^3yz^2)$$

$$= \hat{i}\frac{\partial}{\partial x}(2x^3yz^2) + \hat{j}\frac{\partial}{\partial y}(2x^3yz^2) + \hat{k}\frac{\partial}{\partial z}(2x^3yz^2).$$

$$\vec{\nabla}\phi = 6x^2yz^2\hat{i} + 2x^3z^2\hat{j} + (2x^3y2z)\hat{k}$$

$$(4x^3yz)$$

$$\text{div(grad}\phi) = \vec{\nabla}\cdot(\vec{\nabla}\phi).$$

$$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot\left[(6x^2yz^2)\hat{i} + 2x^3z^2\hat{j} + (2x^3y2z)\hat{k}\right].$$

$$4x^3yz$$

$$= \frac{\partial}{\partial x}(6x^2yz^2) + \frac{\partial}{\partial y}(2x^3z^2) + \frac{\partial}{\partial z}(4x^3yz)$$

div(grad φ) = ∇.∇φ



$$= 12xyz^2 + 0 + 4x^3y.$$

div. (grad φ) $= 12xyz^2 + 4x^3y$ Ans

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Examp 22.

$$\vec{\nabla}.\vec{\nabla}\phi = \nabla^2\phi.$$

$$\vec{\nabla}.\vec{\nabla}\phi = \vec{\nabla}.\left[\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right]\phi.$$

$$= \vec{\nabla}.\left[\hat{i}\frac{d}{dx}\phi + \hat{j}\frac{d}{dy}\phi + \hat{k}\frac{d}{dz}\phi\right].$$

$$= \left[\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right].\left[\hat{i}\frac{d\phi}{dx} + \hat{j}\frac{d\phi}{dy} + \hat{k}\frac{d\phi}{dz}\right]$$

$$= \frac{d}{dx}\left(\frac{d\phi}{dx}\right) + \frac{d}{dy}\left(\frac{d\phi}{dy}\right) + \frac{d}{dz}\left(\frac{d\phi}{dz}\right)$$

$$= \frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dy^2} + \frac{d^2\phi}{dz^2}.$$

$$= \phi\left[\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right].$$

$$\vec{\nabla}.\vec{\nabla}\phi = \nabla^2\phi.$$

$$\boxed{\nabla^2 = \left[\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right]}$$ is called Laplacian operator.

Examp. 23

$\vec{A} = 3y^2z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$ is called a Solenoidal vector.

Sol→ The vector $\vec{A}$ will be the solenoid of vector if,
div $\vec{A} = 0$ or $\vec{\nabla}\cdot\vec{A} = 0$.

33 div. $\vec{A} = \vec{\nabla}\cdot\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left[3y^2z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}\right]$

$= \frac{\partial}{\partial x}(3y^2z^2) + \frac{\partial}{\partial y}(4x^3z^2) + \frac{\partial}{\partial z}(-3x^2y^2)$.

$= \frac{\partial}{\partial x}(3y^2z^2) + \frac{\partial}{\partial y}(4x^3z^2) - \frac{\partial}{\partial z}(3x^2y^2)$.

$= 0 + 0 - 0 = 0$.

$\boxed{\vec{\nabla}\cdot\vec{A} = 0}$.

Examp. 24

$\vec{F} = 2x^2\hat{i} + (4y^2+z^2)\hat{j} - 3yz\hat{k}$.

Sol⇒ $\vec{F} = 2x^2\hat{i} + (4y^2+z^2)\hat{j} - 3yz\hat{k}$

$\vec{\nabla}\cdot\vec{F} = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot\left[(2x^2)\hat{i} + (4y^2+z^2)\hat{j} - 3yz\hat{k}\right]$

$= \frac{\partial}{\partial x}(2x^2) + \frac{\partial}{\partial y}(4y^2z^2) - \frac{\partial}{\partial z}(3yz)$

$= 4x + 8y - 3y \boxed{= 4x + 5y}$

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Example-25.

Sol: The vector field $\vec{A}$ will be the Solenoidal if

$\vec{\nabla}.\vec{A} = 0$.

$\vec{A} = x^2\hat{i} + (y-2xy)\hat{j} + (x+bz)\hat{k}$.

Solenoid $\vec{\nabla}.\vec{A} = 0$

$\vec{\nabla}.\vec{A} = \left[\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right].\left[x^2\hat{i} + (y-2xy)\hat{j} + (x+bz)\hat{k}\right] = 0$.

$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dy}(y-2xy) + \frac{d}{dz}(x+bz) = 0$

$\Rightarrow 2x + (1-2x) + b = 0$.

$\Rightarrow 2x + 1 - 2x + b = 0$

$\Rightarrow 1 + b = 0 \Rightarrow \boxed{b = -1}$

Example-26.

$\phi = x^2 + 2y - z^2$.

Find div. grad. $\phi$.

Grad. $\phi = \vec{\nabla}.\phi = \left[\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right]\left[x^2 + 2y - z^2\right]$

$= \hat{i}\frac{d}{dx}(x^2+2y-z^2) + \hat{j}\frac{d}{dy}(x^2+2y-z^2) + \hat{k}\frac{d}{dz}\left[x^2+2y-z^2\right]$.

div grad of $\phi$ $= \vec{\nabla}.$ $2x\hat{i} + 2\hat{j} + 2z\hat{k}$

div. (grad of $\phi$) $= \vec{\nabla}.\vec{\nabla}\phi$

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$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \cdot \left[2x\hat{i} + 2\hat{j} - 2z\hat{k}\right]$

$= \frac{\partial}{\partial x}(2x) + \frac{\partial}{\partial y}(2\hat{j}) - \frac{\partial}{\partial z}(+2z).$

div (grad ϕ) (Imp) $\cancel{2} + 0 - \cancel{2} = 0$. Ans

Ex. 27.

Sol ⇒ $\vec{A} = (x+3y)\hat{i} + (2y+3z)\hat{j} + (x+az)\hat{k}$

is a Solenoid vector.

$\nabla \cdot \vec{A} = 0$

$\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \cdot \left[(x+3y)\hat{i} + (2y+3z)\hat{j} + (x+az)\hat{k}\right] = 0$

$\Rightarrow \frac{\partial}{\partial x}(x+3y) + \frac{\partial}{\partial y}(2y+3z) + \frac{\partial}{\partial z}(x+az) = 0$

$\Rightarrow 1 + 2 + a = 0$

$\Rightarrow 3 + a = 0 \Rightarrow \boxed{a = -3}$ Ans

Examp. 28.

Sol ⇒ $F_x = x+y$; $F_y = x+y$; $F_z = -2z$.

Here the vector field, $\vec{F} = \left[F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\right]$.

divergence $\vec{F} = \left[(x+y)\hat{i} + (x+y)\hat{j} + (-2z)\hat{k}\right]$.

$\nabla \cdot \vec{F} = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \cdot \left[(x+y)\hat{i} + (x+y)\hat{j} - 2z\hat{k}\right]$

$= \frac{\partial}{\partial x}(x+y) + \frac{\partial}{\partial y}(x+y) + \frac{\partial}{\partial z}(-2z)$

$= 1 + 1 - 2 = \cancel{2} - \cancel{2} = 0$ Ans

Exam 29.

Sol⇒ $\vec{x} = 0$, $\vec{\nabla} \times \vec{x} = 0$. $\vec{x}$ is the p.v. of $\vec{x}$
$= x\hat{i} + y\hat{j} + z\hat{k}$.

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Curl of $\vec{x} = \vec{\nabla} \times \vec{x}$.

$= \left[\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right] \times (x\hat{i} + y\hat{j} + z\hat{k}]$.

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ x & y & z \end{vmatrix} = \hat{i}\begin{vmatrix} d/dy & d/dz \\ y & z \end{vmatrix} - \hat{j}\begin{vmatrix} d/dx & d/dz \\ x & z \end{vmatrix} + \hat{k}\begin{vmatrix} d/dx & d/dy \\ x & y \end{vmatrix}$.

$= \hat{i}\left\{\frac{d}{dy}z - \frac{d}{dz}y\right] - \hat{j}\left(\frac{d}{dx}z - \frac{d}{dz}x\right] + \hat{k}\left[\frac{d}{dx}y - \frac{d}{dy}x\right] = 0$.

Since, $\vec{\nabla} \times \vec{x} = 0$ (Ans.

Examp -30.

Sol⇒ $\vec{A} = xy^2\hat{i} + 2xyz\hat{j} + 2xyz^3\hat{k}$.
at point $(1, -1, -1)$.

Curl $\vec{A} = \vec{\nabla} \times \vec{A}$

$= \left(\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}\right) \times \left[xy^2\hat{i} + 2xyz\hat{j} + 2xyz^3\hat{k}\right]$.

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$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz^2 & 2xyz & 2xyz^3 \end{vmatrix} = \hat{i}\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz & 2xyz^3 \end{vmatrix} - \hat{j}$$

$$\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ xz^2 & 2xyz^3 \end{vmatrix} + \hat{k}\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ xz^2 & 2xyz \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(2xyz^3) - \frac{\partial}{\partial z}(2xyz)\right] + \hat{j}\left[\frac{\partial}{\partial z}(2xyz^3) - \frac{\partial}{\partial z}(xz^2)\right]$$

$$+ \hat{k}\left[\frac{\partial}{\partial x}(2xyz) - \frac{\partial}{\partial y}(xz^2)\right].$$

$$= \hat{i}\left[2xz^3 - 2xy\right] + \hat{j}\left[2xz - 2yz^3\right] + \hat{k}\left[2yz - 0\right].$$

At point $(1,-1,-1)$ we get.

$$\text{curl of } \vec{A} = \vec{\nabla}\times\vec{A} = \hat{i}[0] + \hat{j}[-4] + \hat{k}[2]$$

$$= -4\hat{j} + 2\hat{k} \text{ / Ans}$$

Ex. 31.

curl of $k\vec{A} = k \text{ curl } \vec{A} - \vec{A}\times \text{grad } k$.

$$\text{curl } k\vec{A} = \vec{\nabla}\times k\vec{A}.$$

$$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\times\left[kA_x\hat{i} + kA_y\hat{j} + kA_z\hat{k}\right]$$

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Example-32.

$\vec{\nabla} \times \vec{\nabla} u = 0$.

curl of grad is zero.

$$\vec{\nabla} \times \vec{\nabla} u = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left[\hat{i}\frac{\partial u}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + \hat{k}\frac{\partial u}{\partial z}\right].$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \end{vmatrix}.$$

$$= \hat{i}\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \end{vmatrix} + \hat{j}\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ \frac{\partial u}{\partial x} & \frac{\partial u}{\partial z} \end{vmatrix} + \hat{k}\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial z}\right) - \frac{\partial}{\partial z}\left[\frac{\partial u}{\partial y}\right]\right] + \hat{j}\left(\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}\right)\right)$$

$$+ \hat{k}\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) - \frac{\partial}{\partial y}\left[\frac{\partial u}{\partial x}\right]\right].$$

$$= \hat{i}[0] + \hat{j}[0] + \hat{k}[0] = 0.$$ Ans.

Exam. 33.

curl $|\vec{x}|^n \vec{x} = 0$.

$\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$. is the p.v.

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Sol ⇒ $\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{x}| = (x^2+y^2+z^2)^{1/2}$

$|\vec{x}|^n = (x^2+y^2+z^2)^{n/2}$

$\text{Curl}\,(|\vec{x}|^n \vec{x}) = \nabla \times |\vec{x}|^n \vec{x}$

$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times |\vec{x}|^n (x\hat{i} + y\hat{j} + z\hat{k})$

$\nabla \times |\vec{x}|^n \vec{x} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ |\vec{x}|^n x & |\vec{x}|^n y & |\vec{x}|^n z \end{vmatrix}$

$= \hat{i}\left[\frac{\partial}{\partial y}|\vec{x}|^n z - \frac{\partial}{\partial z}|\vec{x}|^n y\right] + \hat{j}\left[\frac{\partial}{\partial z}|\vec{x}|^n x - \frac{\partial}{\partial x}|\vec{x}|^n z\right]$

$+ \hat{k}\left[\frac{\partial}{\partial x}|\vec{x}|^n y - \frac{\partial}{\partial y}|\vec{x}|^n x\right]$

$= \hat{i}\left[\frac{\partial}{\partial y}(x^2+y^2+z^2)^{n/2} z - \frac{\partial}{\partial z}(x^2+y^2+z^2)^{n/2} y\right]$

$+ \hat{j}\left[\frac{\partial}{\partial z}(x^2+y^2+z^2)^{n/2} x - \frac{\partial}{\partial x}(x^2+y^2+z^2)^{n/2} z\right]$

$+ \hat{k}\left[\frac{\partial}{\partial x}(x^2+y^2+z^2)^{n/2} y - \frac{\partial}{\partial y}(x^2+y^2+z^2)^{n/2} x\right]$

$= \hat{i}\left[n(x^2+y^2+z^2)^{n/2-1} \cdot yz - n(x^2+y^2+z^2)^{n/2-1} yz\right] +$

$\hat{j}\left[n(x^2+y^2+z^2)^{n/2-1} xz - n(x^2+y^2+z^2)^{n/2-1} xz\right] +$

$\hat{k}\left[n(x^2+y^2+z^2)^{n/2-1} xy - n(x^2+y^2+z^2)^{n/2-1} xy\right]$

$= 0.$

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Exam. 34.

$$\text{curl}\ (\vec{A}\times\vec{r}) = \vec{\nabla}\times(\vec{A}\times\vec{r}).$$

$$= \left[\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right]\times\left[A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right]\times\left[x\hat{i}+y\hat{j}+z\hat{k}\right]$$

$$= \left[\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right]\times\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ x & y & z\end{vmatrix}$$

$$= \left[\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right]\times\left[\hat{i}(zA_y-yA_z)+\hat{j}(xA_z-zA_x)+\hat{k}(yA_x-xA_y)\right].$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ (zA_y-yA_z) & (xA_z-zA_x) & (yA_x-xA_y)\end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(yA_x-xA_y)-\frac{\partial}{\partial z}(xA_z-zA_x)\right]+\hat{j}\left[\frac{\partial}{\partial z}(zA_y-yA_z)-\frac{\partial}{\partial x}(yA_x-xA_y)\right]+\hat{k}\left[\frac{\partial}{\partial x}(xA_z-zA_x)-\frac{\partial}{\partial y}(zA_y-yA_z)\right].$$

$$= \hat{i}\left[A_x+A_x\right]+\hat{j}\left[A_y+A_y\right]+\hat{k}\left[A_z+A_z\right].$$

$$= 2A_x\hat{i}+2A_y\hat{j}+2A_z\hat{k}$$

$$= 2\left[A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right] = 2\vec{A}.$$

Curl $(\vec{A} \times \vec{r}) = 2\vec{A}$.

Examp. 40.

$\vec{E}$ will be irrotational field if $\vec{\nabla} \times \vec{E} =$

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$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy & (3x^2 - 3y^2) & 0 \end{vmatrix}.$$

$$= \left[0 - \frac{\partial}{\partial z}(3x^2 - 3y^2)\right]\hat{i} + \left[\frac{\partial}{\partial z}(6xy) - \right.$$

$$\left[\frac{\partial}{\partial x}(3x^2 - 3y^2) - \frac{\partial}{\partial y}(6xy)\right]\hat{k}.$$

$$= 0\hat{i} + 0\hat{j} + [6x - 6x]\hat{k}$$

$$= 0 \quad \text{Ans}$$

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Physics:

Q1 Define Scalar or dot products of two vectors.

Ans: The scalar or dot product of two vectors $\vec{A}$ and $\vec{B}$ is $\vec{A}\cdot\vec{B} = AB\cos\theta$, A is the magnitude of $\vec{A}$ and $\vec{B}$ is the magnitude of $\vec{B}$ and $\theta$ is the angle betn these two vectors.

The scalar or dot product of two vectors gives the scalar physical quantity

Q2 Write the condn for two vectors $\vec{A}$ and $\vec{B}$ to be ⊥lar to each other.

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Ans: $\vec{A}\cdot\vec{B} = AB\cos\theta$, $\vec{A}$ and $\vec{B}$ are ⊥lar to each other

Now, $\theta = 90^\circ$, $\cos\theta = 0 \Rightarrow \cos 90^\circ = 0$,

So, $\vec{A}\cdot\vec{B} = AB\cos 90^\circ = 0$., If two vectors are ⊥lar to each other, their dot product is zero.

Q3 If $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$, show that vectors $\vec{A}$ and $\vec{B}$ are ⊥lar to each other.

Ans:- $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

$|\vec{A}+\vec{B}|^2 = |\vec{A}-\vec{B}|^2$, Now, $\vec{A}\cdot\vec{A} = A^2 = |\vec{A}|^2$

So, $(\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B}) = (\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B})$.

$\vec{A}\cdot\vec{A}+\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{A}+\vec{B}\cdot\vec{B} = \vec{A}\cdot\vec{A}-\vec{A}\cdot\vec{B}-\vec{B}\cdot\vec{A}+\vec{B}\cdot\vec{B}$

$\cancel{\vec{A}\cdot\vec{A}}+\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{A}+\cancel{\vec{B}\cdot\vec{B}}-\cancel{\vec{A}\cdot\vec{A}}+\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{A}-\cancel{\vec{B}\cdot\vec{B}} = 0.$

$4\vec{A}\cdot\vec{B} = 0$, $4 \neq 0$

$\Rightarrow \boxed{\vec{A}\cdot\vec{B} = 0}$, for condn $\vec{A}$ and $\vec{B}$ is zero.

$\vec{A}\cdot\vec{B} = 0$ ; vec... perpendicular to each other).

Q4 Define vector or cross product of two vectors?

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Ans: The vector or cross product of two non-zero vectors is given by $\vec{A}$ and $\vec{B}$ is
$\vec{A}\times\vec{B} = (AB\sin\theta)\,\hat{n}$, Where A and B are the magnitudes of $\vec{A}$ and $\vec{B}$, $\hat{n}$ is the unit vector ⊥ar to the plane containing vector $\vec{A}$ and $\vec{B}$.

Q5 Write the condn. for two vectors $\vec{A}$ and $\vec{B}$ be ∥ to each other.

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Ans:- The two vectors $\vec{A}$ and $\vec{B}$ are ∥ to each other, $\theta = 0°$, $\vec{A}\times\vec{B} = (AB\sin 0°)\,\hat{n} = 0$, if $\vec{A}$ and $\vec{B}$ are ∥ to each other then their cross product is zero.
($\sin 0° = 0$)

Q6 $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the orthogonal unit vectors. Write the values of (i) $\hat{i}\cdot\hat{i}$, $\hat{j}\cdot\hat{j}$ and $\hat{k}\cdot\hat{k}$ (ii) $\hat{i}\cdot\hat{j}$ and $\hat{j}\cdot\hat{k}$, (iii) $\hat{i}\cdot\hat{j}$, $\hat{j}\cdot\hat{k}$ and $\hat{k}\cdot\hat{i}$.

Ans:- Given $\hat{i}$, $\hat{j}$ and $\hat{k}$ are mutually ⊥ar to each other.

(i) $\hat{i}\cdot\hat{i} = 1\times1\times\cos 0° = 1$.
$\hat{j}\cdot\hat{j} = 1\times1\times\cos 0° = 1$
$\hat{k}\cdot\hat{k} = 1\times1\times\cos 0° = 1$

(ii) $\hat{i}\cdot\hat{j} = 1\times1\times\cos 90° = 0$
$\hat{j}\cdot\hat{k} = 1\times1\times\cos 90° = 0$

(iii) $\hat{i}\cdot\hat{j} = (1\times1\times\sin 90°)\,\hat{k} = \hat{k}$ ; $\hat{j}\cdot\hat{k} = (1\times1\times\sin 90°)\,\hat{i} = \hat{i}$

... $= \hat{i}$

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Q8. Distinguish betn. Scalar and Vector fields.

Ans:- Scalar field: The region of space in which the ~~scalar~~ physical quantity has unique value at every point in the space is called Scalar field. Ex.:- Density field, temp. field etc.

The region of space in which the vector physical quantity has unique value at every point in the space are called Vector field. Eg: Electric field intensity, Magnetic field intensity etc.

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Q9. What does gradient of scalar functions represent

Ans:- The gradient of scalar functn. of ($\nabla \phi$) is defined that $max^m$ rate of change of function and is directed along the directn. in which the rate of change of $\phi$ is $max^m$.

Q10. What do you understand by line integral?

Ans:- The integration of vector field along a curve is known as line integral.

Q11. What do you understand by Conservative field or lamellar field?

Ans:- Conservative/lamellar field:- A vector field

whose line ... path is zero is called
Conservative field / lamellar field.
∴ Vector field $\vec{E}$ is said to be conservative.

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Now, $\oint \vec{E}\cdot d\vec{l} = 0$. Ans.

Q12* Define div. of vector field. What is the physical meaning of div. of vector field?

45 Ans: Div. of vector field is the amount of flux flowing per unit Vol$^m$. If div. of vector field is positive at given point, then the fluid is Expanding or its density at that point decrease.

If div. of vector field $\vec{A}$ is -ve. Then the fluid is contracting or its density increase.

If $\vec{A} = 0$ Flux entering any element of the space, neither the fluid Expands nor contracts,

Q13* What do you understand by Solenoid Vector?

Ans: A vector is said to be div. of vector is zero. Let $\vec{A}$ be vector, it will be solenoid, if div. $\vec{A} = 0$

Q14* What is irrotational field?

Ans:- Irrotational field:- A Field whose curl is zero is called Irrotational field $\vec{\nabla}\times\vec{A} = 0$
$\vec{A}$ is called irrotational field.

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Q18 State Gauss's div. theorem?

Ans:- Gauss's Div. theorem: The surface integral of vector field $\vec{A}$ around a closed surface is equal to Volm. integral of vector field divergence of vector field over a Volm. Enclosed by the closed surface.

$$\oiint_S \vec{A}\cdot d\vec{s} = \iiint_V (\vec{\nabla}\cdot\vec{A})\,dv$$

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Q19 State Stoke's theorem?

Ans:- The line integral of vector field ($\vec{A}$) around any closed path is equal to the surface integral of curl of vector field taken over any surface of which the curve is bounding edge.

$$\oint_C \vec{A}\cdot d\vec{l} = \iint_S (\vec{\nabla}\times\vec{A})\cdot d\vec{s}$$

Q20 What is scalar and vector quantity?

Ans:- Scalar quantity:- A physical quantity which is completely specified by its magnitude is called scalar quantity. Ex: Mass, length, time, Volume etc

Vector quantity:- A physical quantity which is completely specified by its magnitude and direction its space is called Vector quantity.

Ex:- Displa. velocity, acce-leration, force etc.

Unit Vector: A vector of unit magnitude and whose direction is same as that of given vector is called Unit Vector. $A = |\vec{A}|\hat{A}$ = $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$

$A = |\vec{A}|\hat{A}$
$\hat{A} = \frac{|\vec{A}|}{A}$

5) Curl of vector: The cross product of $\vec{\nabla}$ and vector field $\vec{A}$ is called curl of vector.

$$\vec{\nabla} \times \vec{A}.$$

**Gradient of Scalar field:** When scalar field ($\phi$) is operated upon by a del operator $\vec{\nabla}$, we get a vector function which is called Gradient of scalar field.

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$$\text{grad.}\,\phi = \vec{\nabla}\phi = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right]\phi.$$

$$= \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}.$$ Grad. $\phi$ is a vector fund.

Ex. of Grad. of scalar funct.

(i) Gravitational force: $\vec{F} = -\vec{\nabla} U$ { U is G. Potent. energy}.

(ii) Electric field due to static charge distri.

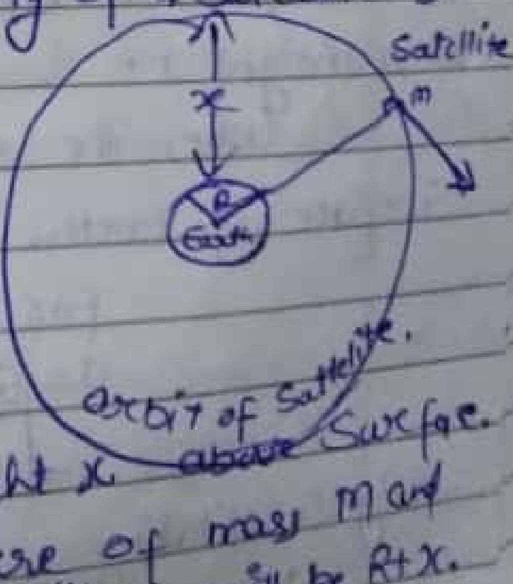
$$\vec{F} = -\vec{\nabla} V$$

{ V is Electric Potent. energy.

Mechanics:

Orbital Velocity and the time period of satellite:

(a) Orbital Velocity: Orbital velocity of a satellite is the velocity required to put a satellite into its orbit around the Earth.




Suppose that a satellite of mass m has to put into circular orbit around the Earth at height x above surface. and consider that Earth is a sphere of mass M and Radius R. The radius of orbit of the satellite will be R+x. Suppose that v is the req. orbital velocity

$\frac{(x^2+y^2)dx}{x^2+y^2} + \frac{2x\,dx + 2y\,dy}{x^2+y^2} = 0$

$dx + d\left[\log(x^2+y^2)\right] = 0$

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$d\left[x + \log(x^2+y^2)\right] = 0$

Int. we get :—

$x + \log(x^2+y^2) = C$ which is req. solⁿ. Ans

Physics :—

1. If $\vec{A}\cdot\vec{B} = 0$.

Ans :— $\vec{A}$ and $\vec{B}$ are ⊥lar to each other.

2. Vectors $\vec{A}$ and $\vec{B}$ lie in x-y plane and $\vec{A}\times\vec{B} = AB(\sin\theta)\hat{n}$. The direction of $\hat{n}$ is along

(i) +ve x-axis (ii) +y-axis.

(iii) +ve z-axis (iv) -ve z-axis.

Ans :— +ve z-axis.

3. The vectors $\vec{A}$ and $\vec{B}$ such that $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$. The angle betⁿ $\vec{A}$ and $\vec{B}$ is

(i) 30° (ii) 60° (iii) 75° (iv) 90°.

Ans :— 90° {i.e $\vec{A}$ and $\vec{B}$ are ⊥lar to each other}.

4. The angle betⁿ $\vec{A} = 5\hat{i} - 5\hat{j}$ and $\vec{B} = 5\hat{i} + 5\hat{j}$ is:

(i) 0° (ii) 45° (iii) 60° (iv) 90°.

Ans :— 0° Ans

$|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

$|5\hat{i}-5\hat{j}+5\hat{i}+5\hat{j}| =$

$\Rightarrow |5\hat{i}-5\hat{j}+5\hat{i}+5\hat{j}| \quad |5\hat{i}-5\hat{j}-(5\hat{i}+5\hat{j})|$

$|5\hat{i}-5\hat{j}-5\hat{i}+5\hat{j}|$

$\Rightarrow |5\hat{i}-5\hat{j}+5\hat{i}+5\hat{j}| = |5\hat{i}-5\hat{j}-5\hat{i}-5\hat{j}|$

$5\hat{i}+5\hat{j}+5\hat{i}+5\hat{j}$

$10\hat{i}+10\hat{j}$

Q5 The angle betⁿ $(\hat{i}+\hat{j})$ and $(\hat{j}+\hat{i})$ is (where $(\hat{i}$ and $\hat{j})$ are unit vectors along x-axis and y-axis.
(i) $30^\circ$ (ii) $45^\circ$ (iii) $0^\circ$ (iv) $90^\circ$

Ans:— $0^\circ$

Q6 If the dot and cross products of two vectors $\vec{A}$ and $\vec{B}$ are equal in magnitude, then the angle betⁿ vectors $\vec{A}$ and $\vec{B}$ is.
(i) $45^\circ$ (ii) $90^\circ$ (iii) $120^\circ$ (iv) $180^\circ$.

Ans:— $45^\circ$

Q7 Directⁿ of grad. $\phi$ is.
(i) Always normal to the surface $\phi$ = Constant.
(ii) Always parallel " " " = "
(iii) Normal to ∥ to the surface depending upon its shape.
(iv) None of these.

Ans:— Always normal to the surface $\phi$ = Constt

Q8 For a Conservative or lamellar field.
(i) $\oint \vec{E}\cdot d\vec{l} = E\,l$ (ii) $\oint \vec{E}\cdot d\vec{l} = 1$ (iii) $\oint \vec{E}\cdot d\vec{l} = 0$
(iv) $\oint \vec{E}\cdot d\vec{l} = E\,dl\cos\theta$

Ans:— $\oint \vec{E}\cdot d\vec{l} = 0$ (Acc. to defⁿ. of c.f.).

Q9. A vector $\vec{v}$ is solenoidal if.
(i) $\vec{\nabla}\times\vec{v} = 0$ (ii) $\vec{\nabla}\cdot\vec{v} = 0$ (iii) $\vec{\nabla}\times\vec{A} = 1$ (iv) $\vec{\nabla}\cdot\vec{A} = 1$.

Ans:— $\vec{\nabla}\cdot\vec{v} = 0$ Ans.

Q10. A vector field $\vec{A}$ is irrotational if:
(i) $\vec{\nabla}\times\vec{A}=0$ (ii) $\vec{\nabla}\cdot\vec{A}=0$ (iii) $\vec{\nabla}\times\vec{A}=1$
(iv) $\vec{\nabla}\cdot\vec{A}=1$.

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Ans:— $\vec{\nabla}\times\vec{A}=0$.

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Q11. Which type of field can be expressed as gradient of scalar field function?

Ans:— Lamellar Field.

Q12. A vector field is said to be conservative if it can always be written as
(i) $\vec{A}=\vec{\nabla}\cdot U$ (ii) $\vec{A}=\vec{\nabla}\times U$ (iii) $\vec{A}=-\vec{\nabla}V$ (iv) $\vec{A}=\nabla^2 U$.
Ans:— $\vec{A}=-\vec{\nabla}V$.

Q13. Which type of field can be expressed as gradient scalar function?
Ans:— Solenoid.

Q14. The div. of curl of vector is always.
Ans:— Zero.

Q15. The direction of $\vec{\nabla}\phi$ ($\phi$ being const.) is always,
(i) ⊥lar to surface
(ii) ∥ to the "
(iii) ⊥lar or ∥ to the surface
Ans:— ⊥lar to the surface.

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Q16: Show that curl $\vec{r} = 0$, where
$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

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Solution:- curl $\vec{r} = \vec{\nabla} \times \vec{r}$.

$$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \times \left[x\hat{i} + y\hat{j} + z\hat{k}\right].$$

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$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix} = \hat{i}\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ y & z \end{vmatrix} - \hat{j}\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x & z \end{vmatrix} + \hat{k}\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x & y \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right] + \hat{j}\left[\frac{\partial z}{\partial x} - \frac{\partial x}{\partial z}\right] + \hat{k}\left[\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right].$$

$= 0$ Ans.

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# CHAPTER-2

# Electrostatics

## Coulomb's Law :— Charles A. Coulomb

gave a law to determine the force of attraction or force of repulsion betn. two point charges at rest.
The force betn. two point charges at rest is called **Electrostatic force or Coulomb's force**

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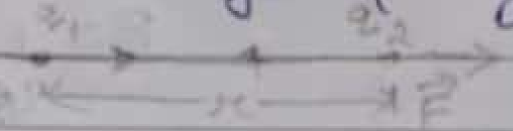
- Actual definition is the force or repulsion betn. (i) two point charges is directly proportional to the product of magnitude of the charges and (ii) inversely proportional to the square of distn. betn. them.

Let us consider two like point charges of mag. $q_1$ and $q_2$ seperated by distn. $x$. The force of attraction and repulsion betn them is given by:-




$$F \propto \text{(i) } q_1 q_2$$
$$\text{(ii) } 1/x^2 \Rightarrow F \propto \frac{q_1 q_2}{x^2}$$

$\Rightarrow F = K \frac{q_1 q_2}{x^2}$ where $K$ is constt. of proportionality.

In vector form, we may write:

$\vec{F} = K \frac{q_1 q_2}{|x|^2} \hat{x}$. , $\hat{x}$ is the unit vector in the directn. from $q_1$ to $q_2$.

$\vec{F}$ is the force on the charge $q_2$ due to charge $q_1$. The force on charge $q_1$ due to charge $q_2$ is equal and opposite to $\vec{F}$.

- Value of K depends upon the choice of syst and nature of the medium.

53 CGS system, $K = \frac{1}{K}$, K is the dielectric con of medium.

$F = \frac{1}{K}\frac{q_1 q_2}{r^2}$, for vaccum and air, $K = 1$

$\Rightarrow F_0 = \frac{q_1 q_2}{r^2}$ $\Rightarrow K = \frac{1}{4\pi\epsilon_0\epsilon_r}$, $\epsilon_0 = 8.85 \times 10^{-12}$

and is called permitivity of free space and $\epsilon_r$ is called the relative permittivity of the medium.

$$F = \frac{1}{4\pi\epsilon_0\epsilon_r}\frac{q_1 q_2}{r^2}$$

For air and Vaccum $\epsilon_r = 1$.

$$\Rightarrow F_0 = 9\times10^9 \frac{q_1 q_2}{r^2} \text{ Ans.}$$

## Units of charge:-

**In CGS sys.:** Unit of Electric charge is … or stat…

**In SI syst.:** " " " is Coulomb, … by C…

units of charge:- C.G.S 1 e.s.u.
SI Coulomb (C).

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**Electric Field:-** The space or region around a charged body, electric influence (like force of attraction and repulsion) can be experienced by another charge is called Electric field.

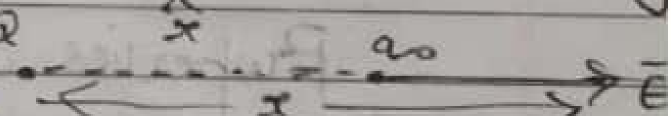
**Electric field vector:-** Electric field vector at a point in the space, the Electrostatic force can be experienced by the unit positive test charge called Electric field vector.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

unit of Electric field:- $NC^{-1}$.

⇒ Electric field due to a point source charge

Consider a point source charge +Q which creates the Electric field in the source charge. On the other hand $q_0$ is the test charge & placed in the Electric field at dispn. $x$.




$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0}{x^2}\hat{x}$$

$\hat{x}$ is the unit vector is directed from point source charge +Q to test charge $q_0$.

Vector

Electric field due to point source charge is,

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{q_0} \frac{1}{4\pi\varepsilon_0} \frac{Qq_0 \hat{r}}{r^2}$$

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$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q\hat{r}}{r^2}}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

Electric field due to point source charge is directly proportional to the magnitude of the source charge and inversely proportional to the square of the radius of point source charge.

## Electric Field lines:—

The concept of Electric field lines and Electric lines of force was introduced by Faraday.

- Electric field lines is a curve that a tangent at any point on it gives the electric field at that point.

## Properties of Electric field lines:—

(i) The field lines is originate from +ve charge and on -ve charge.

(ii) The tangent to a field line at any point on it gives the Electric field at that point.

(iii) Two field lines can't cross each other. This is bec. the intensity of electric field has point a single value. But, If, we have to take two lines are cross each other. Then, there will be two tangents and two values of the intensity of Electric field.

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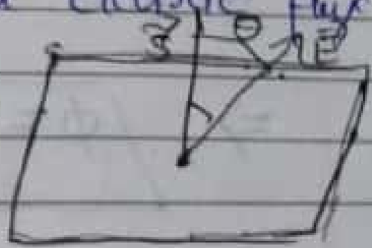
(iii). If, the Electric field lines are very closer then the Electric field is strong. But, on the other hand If, the Electric field lines are far away then the Electric field is so weak.

## Electric Flux ($\phi_E$).

- Electric flux through any surface is defined as that, the no. of field lines of Electric field lines passing through the surface, the surface being ⊥ar to the Electric field called Electric flux.



Consider a surface area $\vec{S}$. Let the Electric field intensity makes an angle $\theta$ with $\vec{S}$.

$$\phi = \vec{E}\cdot\vec{S} = ES\cos\theta \quad \text{--- (1)}$$

Special Case :—

(i) If $\theta = 0^\circ$, The $\vec{E}$ is in the direction of $\vec{S}$. Then, $\phi_E = ES\cos 0^\circ = ES$ (Maximum). The Electric flux through a given surface is something value.

(ii) If $\theta = 90^\circ$, The $\vec{E}$ is ⊥ar to the $\vec{S}$. Then, $\phi_E = ES\cos 90^\circ = 0$, The Electric flux through a given surface is zero.

## Gauss's theorem of Electrostatics (Integral Form)

Gauss's Theorem:- The total Electric flux through a closed surface is equal to $1/\varepsilon_0$ times the total charge Enclosed by the closed surface.

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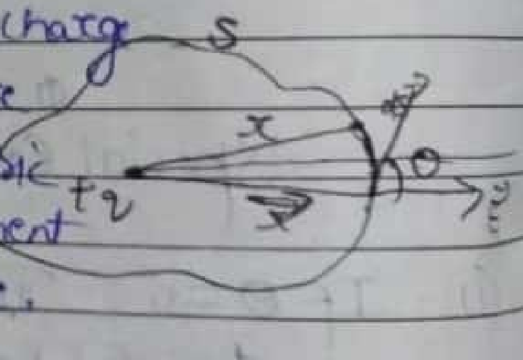
Electric flux $\phi$ through a closed surface is given by :-

$$\phi = \oint\!\!\oint_S \vec{E}\cdot d\vec{s} = q/\varepsilon_0$$

The net charge Enclosed by the closed surface is zero.

$$\Rightarrow \boxed{\phi = \oint\!\!\oint_S \vec{E}\,d\vec{s} = 0}$$

Proof:— Consider a test charge +q inside the surface and radius x and, Electric field intensity of the Element area ds of the surface.




$$\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}\hat{x}$$

$\Rightarrow$ The Electric flux through the area ds is given

$$d\phi = \vec{E}\cdot d\vec{s}. \Rightarrow d\phi = |\vec{E}||d\vec{s}| \Rightarrow \phi = E$$

$$d\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}\hat{x}.\,|d\vec{s}|\cos\theta.$$

$$d\phi = \frac{q}{4\pi\varepsilon_0 x^2} \cdot \frac{|d\vec{s}|\cos\theta}{x^2}$$

$$\oint \text{i.e.} \frac{|d\vec{s}|\cos\theta}{x^2} = d\Omega$$

$$d\phi = \frac{q}{4\pi\varepsilon_0} \cdot \frac{|d\vec{s}|\cos\theta}{x^2} \Rightarrow \phi = \frac{q}{4\pi\varepsilon_0} d\Omega$$

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i.e. $d\Omega$ is the solid angle subtended by the surface.

Now, Total Electric flux through the closed surface is,

$$\boxed{\phi = \oiint_S \vec{E}\cdot d\vec{s} \Rightarrow \phi = \vec{E} \oiint_S d\vec{s}}$$

$$\phi = \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2} \hat{x} \oiint_S d\vec{s}$$

$$\oiint_S d\vec{s} = 4\pi$$

$$\phi = \frac{1}{\cancel{4\pi}\varepsilon_0} \cdot \frac{q}{x^2} \hat{x} \cancel{4\pi}$$

$$\phi = \frac{q}{\varepsilon_0 x^2} \hat{x} \Rightarrow \phi = \frac{q}{\varepsilon_0}$$

$$\phi = \oiint_S \vec{E}\cdot d\vec{s} \Rightarrow \phi = \vec{E} \oiint_S d\vec{s}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 x^2}$$

$$\Rightarrow \phi = \frac{1}{4\pi\varepsilon_0} q \oiint_S d\vec{s}$$

$$\Rightarrow \phi = \frac{q}{\cancel{4\pi}\varepsilon_0} \cancel{4\pi} \Rightarrow \phi = \frac{q}{\varepsilon_0}$$

$$\phi = \oiint_S \vec{E}\, d\vec{s} = \frac{q}{\varepsilon_0}$$

$$\varepsilon_0 \oiint_S \vec{E}\cdot d\vec{s} = q$$

$$\oiint_S \varepsilon_0 \vec{E}\cdot d\vec{s} = q$$

$$\boxed{\oiint_S \vec{D}\, d\vec{s} = q}$$

(i) Outward Electric flux through a closed surface taken as +ve.

(ii) Inward electric flux through a closed surface taken as -ve.

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# Gauss Theorem in Differential Form.

Acc. to Gauss's theorem i.e

$$\oiint_S \vec{E}\cdot d\vec{s} = q/\varepsilon_0 \quad \text{--- (1)}$$

59 When the charge is distributed over a volm. v such that $\rho$ is the volm. density of the charge.

$$q = \iiint_V \rho\, dv \quad \text{--- (2)}$$

$\Rightarrow$ Substituted in Eqn. (1), we get

$$\oiint_S \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}\iiint_V \rho\, dv$$

$$\Rightarrow \varepsilon_0 \oiint_S \vec{E}\cdot d\vec{s} = \iiint_V \rho\, dv \quad \text{--- (3)}$$

Acc. to Gauss's Divergence theorem,

$$\oiint_S \vec{E}\cdot d\vec{s} = \iiint_V (\vec{\nabla}\cdot\vec{E})\, dv \quad \text{--- (4)}$$

put in Eqn. (3),

$$\varepsilon_0 \iiint_V (\vec{\nabla}\cdot\vec{E})\, dv = \iiint_V \rho\, dv$$

$\rightarrow$ Integrands must be Equal to,

$$\varepsilon_0\, \vec{\nabla}\cdot\vec{E} = \rho$$

$\boxed{\vec{\nabla}\cdot\vec{E} = D}$

Displacement current

$$\varepsilon_0 D = \rho$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0}$$

which is the Gauss's theorem differential form

Mag. of accleration is:—

$$a = |\vec{a}| = \sqrt{|\vec{a}_r|^2 + |\vec{a}_\theta|^2 + |\vec{a}_\phi|^2}$$ Ans

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# CHAPTER II

## Solid Angle:—

The ratio of the area of any sphere intercepted by the Cone to the ~~radius of~~ ~~Square~~ square of the radius of that particular sphere is Constt. is called Solid angle.

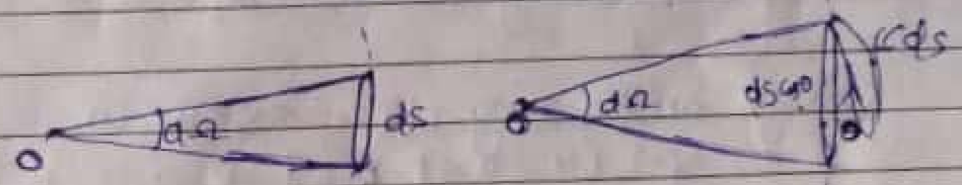
It is denoted by $\Omega$.

i.e. $S$ is the area of surface of radius $r$.

Now,

(i) The solid angle is subtended by area $ds$ at point o at distⁿ $r$.

$$d\Omega = \frac{|d\vec{s}|}{r^2}$$



$\Omega = ds^2$

(ii) The solid angle is subtended by area $ds$ at point o at distⁿ $r$.

Now, $$\Omega = \frac{|d\vec{s}|\cos\theta}{r^2}$$

(iii) The solid angle is subtended at the centre of the closed surface.

$$\Omega = \int d\Omega = \oiint \frac{ds}{r^2} = \frac{1}{r^2}\oiint ds$$

$\oiint ds = 4\pi r^2$

$$\Rightarrow \Omega = \frac{1}{\cancel{r^2}} \times 4\pi \cancel{r^2}$$

$$\Rightarrow \Omega = 4\pi \text{ steradian}$$ Ans

$D = \epsilon_0 E$

$\Rightarrow \vec{D} = \rho/\epsilon_0$

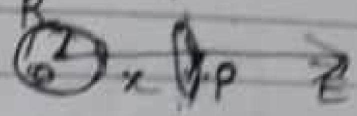
$\Rightarrow \boxed{\vec{\nabla} \cdot \vec{D} = \rho}$

60 Electric flux is directly proportional to the magnitude of the charge.

## Applications of Gauss's Theorem:—

(A) Electric field due to ~~sym~~ spherically symmetric charge distribution.

(i) <u>At a point outside the charged</u> ~~Solid sphere~~:— Consider a charged sphere of a Radius R on which the q is uniformly distributed.




Let point P outside the charged solid sphere on the Gaussian surface whose distⁿ from O to P is x. Then, Now draw an imaginary sphere of radius x with O as centre. The imaginary sphere is called Gaussian sphere. The electric field intensity at every point on the Gaussian surface is same and is directed radially outward.

We assume small element ds in this given surface,

Now, <u>Electric flux</u> over the whole Gaussian surface,

$$d\phi = \vec{E} \cdot d\vec{s} = q/\epsilon_0$$

Total Electric flux over a whole Gaussian Surface is given,

$$\Rightarrow \phi = \oiint \vec{E}\cdot d\vec{s} = q/\varepsilon_0$$

61 i.e $\vec{E}$ and $d\vec{s}$ are parallel to each other) i.e $\theta = 0°$

$$\oiint E\,ds\cos 0° = q/\varepsilon_0$$

$$E \oiint ds = q/\varepsilon_0$$

i.e $\oiint ds$ = Surface area of the Gaussian Surface $= 4\pi x^2$

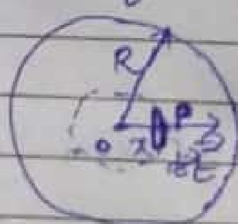
$$\Rightarrow E \times 4\pi x^2 = q/\varepsilon_0$$

$$\Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2}}$$

(b) At a point on the surface of the charged Sphere :—
Electric field intensity can be calculated by substituted
$R = x$.

i.e $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}$

(c) At a point inside the solid charged sphere :—
Consider a solid charged sphere of radius $x$)




$\Rightarrow$ Let point P inside the solid charged sphere whose distⁿ $x$ from O to P. Consider $d\vec{s}$ be the small element in this spherical charged sphere. Draw an imaginary surface w/ centre O to P is $x$.
The imaginary surface is called Gaussian Surface

Let $\rho$ be the charge density of the sphere.

$$\rho = \frac{\text{Charge}}{\text{Vol}^m \text{ of the sphere}} = \frac{q}{\frac{4}{3}\pi R^3}$$

62 charge on the Gaussian Surface is,

$$q' = \rho \times \text{Vol}^m \text{ of the Gaussian Surface.}$$

$$\Rightarrow q' = \rho \times \frac{4}{3}\pi x^3.$$

$$\Rightarrow q' = \frac{q}{\cancel{\frac{4}{3}\pi} R^3} \times \cancel{\rho} \cdot \cancel{\frac{4}{3}\pi} x^3.$$

$$\Rightarrow q' = \frac{q x^3}{R^3}.$$

Now, Acc. to Gauss's theorem

$$\oiint_S \vec{E} \cdot \vec{ds} = \frac{q'}{\varepsilon_0} = \frac{1}{\varepsilon_0} \times \frac{q x^3}{R^3}.$$

Since, $\vec{E}$ and $\vec{ds}$ are parallel $\theta = 0^\circ$.

$$\oiint_S E \cdot ds \cos 0^\circ = q'/\varepsilon_0$$

$$\oiint_S E ds = \frac{1}{\varepsilon_0} \times \frac{q x^3}{R^3}.$$

$$E \oiint_S ds = \frac{1}{\varepsilon_0} \times \frac{q x^{3}}{R^3}.$$

$\oiint_S ds$ = Surface area of the Gaussian Surface

$$E \times 4\pi x^2 = \frac{1}{\varepsilon_0} \times \frac{q x^3}{R^3}.$$

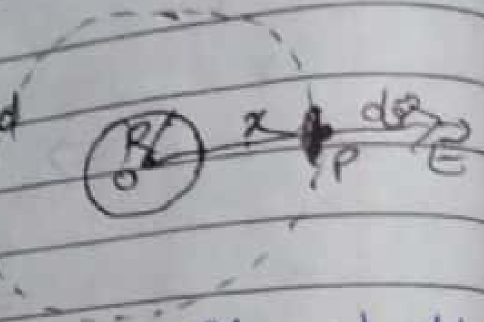
$$E = \frac{1}{4\pi\varepsilon_0 x^2} \cdot \frac{q x^{3}}{R^3}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q x}{R^3}$$

$$\boxed{E \propto x}$$

Electric field intensity inside the charged solid sphere is proportional to the dist$^n$. of the point of observation.

63 Electric field due to charged Spherical shells:—




(a) At a point outside the charged Spherical Shell:— Consider a charged spherical shell a infinitesimally.

Let point P of the charged sphere on the Gaussian Surface. Imaginary sphere x is to from O to P is x. Draw a sphere of radius x from O to P.

Electric field intensity $\vec{E}$ at every point of this sphere.

Acc. to Gauss's theorem

$$\oiint_S \vec{E}.d\vec{s} = q/\varepsilon_0$$

$\vec{E}$ and $\vec{S}$ are parallel.

$$\oiint_S E\,ds = q/\varepsilon_0 \Rightarrow E \oiint_S ds = q/\varepsilon_0$$

$$E \times 4\pi x^2 = q/\varepsilon_0$$

$$E = \frac{1}{4\pi\varepsilon_0} q/x^2$$

Electric field intensity outside the charged spherical shell behaves as if whole charge concentrated at the centre and $\vec{E}$ is directed outward.

(b) At a point on the Surface of Charged Spherical Shell :— Electric field intensity can be Calculated, Substituted $R = x$.

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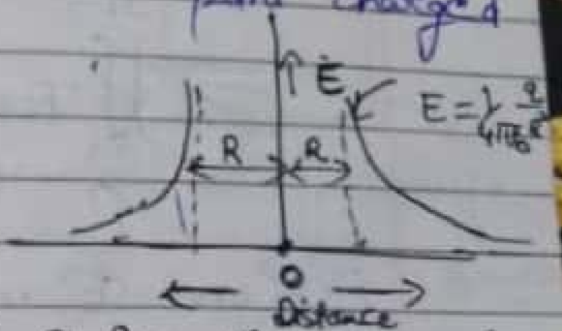
$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}$$

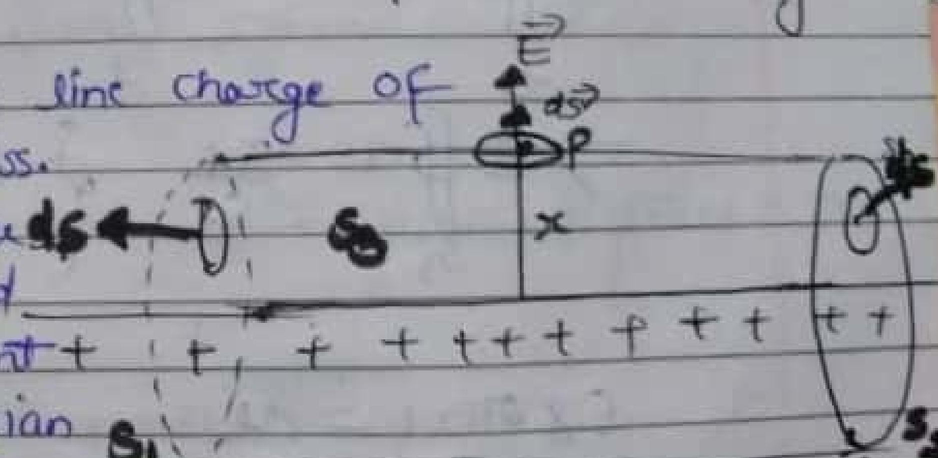
(c) At a point inside the charged spherical shell :— The Gaussian Surface lying inside the shell Enclases no charge. Acc. to Gauss's theorem, an Electric Field Intensity at a point charged spherical shell is zero.

$$E = 0.$$




## ★ Electric field due to infinite line charge :—

- Consider an infinite line charge of negligible thickness. Let Linear Charge density $\lambda$ is placed in air. Let point P is on the Gaussian surface in the form of infinite line charge at distn. $x$ on the wire of the charge.




So, The Gaussian Surface has divided into three Equal parts are $S_1$, $S_2$ and $S_3$.

Acc. to Gauss's theorem:

$$\oint\!\!\oint_S \vec{E}\cdot d\vec{s} = q/\varepsilon_0 = \frac{\lambda l}{\varepsilon_0}$$

Linear charge density $\lambda = \frac{q}{l}$

$q = \lambda l$

Now, $\oint\!\!\oint_{S_1} \vec{E}\cdot d\vec{s} + \oint\!\!\oint_{S_2} \vec{E}\cdot d\vec{s} + \oint\!\!\oint_{S_3} \vec{E}\cdot d\vec{s} = \frac{\lambda l}{\varepsilon_0}$

Now, $\vec{E}$ is ⊥ar to the $d\vec{s}$ for sections $S_1$ and $S_2$,

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$$\Rightarrow \oint\!\!\oint_{S_1} \vec{E}\, ds \cos 90^\circ = 0$$

$$\Rightarrow \oint\!\!\oint_{S_2} E\, ds \cos 90^\circ = 0$$

i.e. $\vec{E}$ and $d\vec{s}$ are parallel in Surface $S_3$ is,

$$\oint\!\!\oint_{S_3} E\, ds \cos 0^\circ = E \oint\!\!\oint_{S_3} ds = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow E \oint\!\!\oint_{S_3} ds = \frac{\lambda l}{\varepsilon_0} \quad \left\{ \text{i.e. } \oint\!\!\oint_{S_3} ds = (\text{Circum} \times \text{length}) = 2\pi x l \right.$$

$$E \times 2\pi x l = \frac{\lambda l}{\varepsilon_0}$$

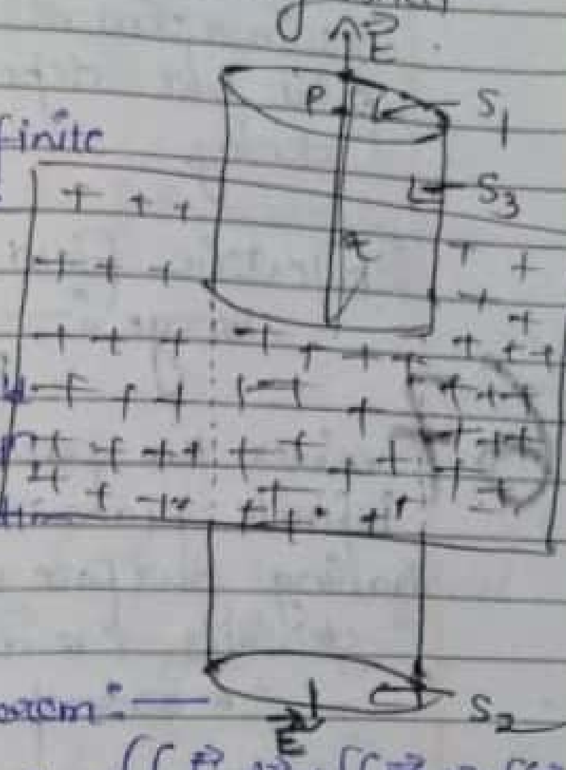
$\frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{x}$

$E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\cancel{l}\lambda}{x\cancel{l}}$, Dividing and multiply of 2,

$$\Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{x}}$$

The direcⁿ of $\vec{E}$ is radially outward from infinite line charge.

• Electric field due to Non-Conducting sheet of charge:—




Consider a non-conducting infinite thin sheet having surface charge density $\sigma$. So,
66 Let P be a point at distn. $r$ and $\vec{E}$ Electric field is radially outward direction, we have three cross-section
Now, we have three cross-section $S_1, S_2$ and $S_3$.

So, Applying Gauss's theorem:—

$$\oiint_S \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0}, \quad \iint_{S_1} \vec{E}\cdot d\vec{s} + \iint_{S_2} \vec{E}\cdot d\vec{s} + \iint_{S_3} \vec{E}\cdot d\vec{s} = \frac{\sigma S}{\varepsilon_0}$$

$\sigma = \frac{q}{S}$, $q = \sigma S$

$\Rightarrow$ $\vec{E}$ and $d\vec{s}$ are $\perp$lar to the $S_3$ surface,

$$\oiint_{S_3} \vec{E}\, d\vec{s} = \frac{\sigma S}{\varepsilon_0}.$$

$$\Rightarrow \iint_{S_3} E\,ds \cos 90^\circ = \cancel{\frac{\sigma S}{\varepsilon_0}} \Rightarrow \iint_{S_3} E\,ds = 0.$$

$\vec{E}$ and $d\vec{s}$ are parallel to $S_1$ and $S_2$.

$$\iint_{S_1} E\,ds \cos 0^\circ + \iint_{S_2} E\,ds \cos 0^\circ = \frac{\sigma S}{\varepsilon_0}$$

$$\Rightarrow E\iint_{S_1} ds + E\iint_{S_2} ds = \frac{\sigma S}{\varepsilon_0}$$

$$ES + ES = \frac{\sigma S}{\varepsilon_0}$$

$$2ES = \frac{\sigma S}{\varepsilon_0}$$

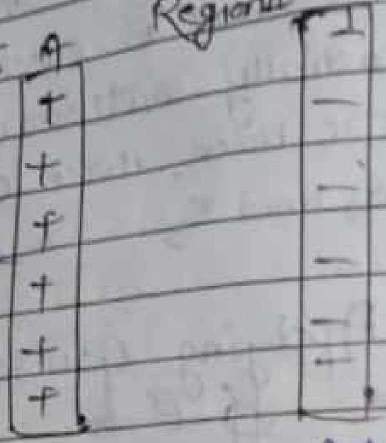
$$E = \frac{\sigma S}{2S\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

• Electric field intensity due to non-conducting charged sheet is independent of the distn. of obs. point but only depends on the surface charge density.

Electric field intensity due to Non-Conducting sheet is independent of the distⁿ of the observation point but dependent of the surface charge density.

* Electric field due to two parallel sheets of charge:-




67 Consider two parallel infinite plane charge having surface charge densities $+\sigma$ and $-\sigma$ etc.

⇒ Electric field intensity due to point A $E = \frac{\sigma}{2\varepsilon}$

The directⁿ of Electric field intensity $E_A$ is away from sheet A.

⇒ Electric field intensity due to point B $E = \frac{-\sigma}{2\varepsilon}$

The -ve sign shows the directⁿ of $E_B$ is toward the sheet B.

Magnitude of $E_A$ = Magnitude of $E_B$

There are three regions I, II, III, I to the left side of sheet A, II region betⁿ the sheet A and sheet B. III region to the right side of the sheet B.

Now, Electric field intensity ~~due~~ the sheet A and B

In the region I, $\vec{E} = E_A(\hat{i}) + E_B(\hat{i})$.

$$= \frac{\sigma}{2\varepsilon_0}(\hat{i}) + \frac{\sigma}{2\varepsilon_0}(\hat{i}) = 0$$

$\Rightarrow$ Electric field intensity due to Both sheets A and B, $\vec{E}$ Region II $= E_A\,\hat{i} + E_B\,\hat{i}$

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$$= \frac{\sigma}{\varepsilon_0}$$

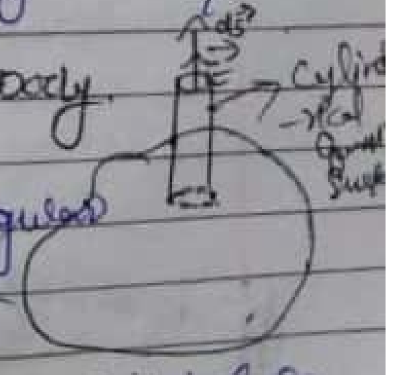
$\Rightarrow$ Electric field intensity due to Both sheets A and B In the region III. $\vec{E} = E_A(\hat{i}) + E_B(\hat{i})$,

$$= \frac{\sigma}{2\varepsilon_0}(\hat{i}) + \frac{\sigma}{2\varepsilon_0}(\hat{i}) = 0$$

Now, Electric Field intensity due to both sheets A and B are Equally and oppositly charged $\vec{E} = \frac{\sigma}{\varepsilon_0}$.

positive charge is in the region I to left side of the region I. in sheet A and region of the -vely Charged Sheet B to the right side is zero.

- Electric Field due to a Charged body.



Consider a Charged Conductor of irregular shape. Let $\sigma$ be the surface charge density. Here we have in sphere the Gaussian surface in the form of Cylinder. Say The Gaussian Surface in the form of Cylinder whose half portion is Embedded in the Cond. and half portion outside the Conductor.

The Gaussian Surface has three parts, two circular Ends ~~of the~~ and one curved portion.
⇒ The Electric Field $\vec{E}$ is ∥ar to the Surface of the Conductor. Electric Field inside the Conductor is Zero. No, Electric Flux is linked with the one circular End of the Gaussian surface.

69 ⇒ The Electric Field Intensity is ∥ar to the one curved portion of the surface of Cond. So, no Electric Flux is linked with one curved portion. But it is when, only when Electric flux is linked circular End of the Gaussian Surface outside the Conductor.

Consider a Small Element of area vector $d\vec{s}$ of the circular End.

⇒ Electric Flux through the small Element is
$$d\phi = \vec{E}\cdot d\vec{s}$$

Total electric flux through the Small Element
$$\phi = \oint\!\!\oint_S \vec{E}\cdot d\vec{s}$$

⇒ Acc. to Gauss's theorem,
$$\oint\!\!\oint_S \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0} = \frac{\sigma S}{\varepsilon_0}$$

$E\,ds\cos 0° = \frac{\sigma S}{\varepsilon_0}$

$$E\oint\!\!\oint_S ds = \frac{\sigma S}{\varepsilon_0}$$

$$E\,S = \frac{\sigma S}{\varepsilon_0}$$

$$E = \frac{\sigma S}{S\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

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• Derivation of Coulomb's law from Gauss's law.

Consider an Isolated charge q. Now imaginary sphere of radius r which is called Gaussian Surface.

Acc. to Gauss's theorem:—

$$\oint_S \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \oint_S E\,ds\cos 0^\circ = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \oint_S E\,ds = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E\oint_S ds = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E \times 4\pi r^2 = \frac{q}{\varepsilon_0} \qquad E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \quad \text{—(i)}$$

⇒ Value of E is Constt for all point on the Gaussian Surface.

In vector form is,

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\,\hat{r}}$$

Q:- Derive units of electric flux.

Ans:— Units of Electric Flux:—

Electric Flux $d\phi = \vec{E}\cdot d\vec{s}$

i.e. Unit of flux $= \vec{E} \times d\vec{s}$

$= NC^{-1} \times m^2$

$\boxed{d\phi = Nm^2C^{-1}}$ Ans.

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# 7| [SHORT ANSWER Type QUESTION].

Q1. State Coulomb's Law for static electric charge. Write the mathematical Expression of Coulomb's Law of the Electric Charge?

Ans ⇒ The force of attraction or repulsion between two point charges at rest is directly to the product of magnitude of the charges and inversely proportional to the square of the distn. betn. two charges. This is Known as Coulomb's Law for static charges.

Expressions of Coulomb's Law is,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} = K \frac{q_1 q_2}{r^2}.$$

F is the Coulomb's forces, $q_1$ and $q_2$ be the any two point charges and $r$ is the distn betn. the any two point charges.

Q2. Define dielectric Constt. of a medium.

Ans ⇒ Dielectric Constt. of a medium is defined that the ratio of Coulomb's force betn. point charges placed at a certain distance apart Vaccum to the Coulomb's force betn. the same two any charges placed at the same distance apart in the vaccum.

Q3. Coulomb's Force between two point charges kept at a distance 'd' apart in air is F. If, the same two point charges separated by a dist are placed in dielectric Const. K, what is the new Coulomb's force betn the two point charges?

Ans ⇒ Acc. to the definition of dielectric Const K is given by,

$$K = \frac{F}{F_m}$$, where, F is Coulomb's force

betn two point charges seperated by distn. d in air and $F_m$ is the ~~charg~~ Coulomb's force betn the same point charges in medium. Coulomb's force bet the point charges in medium is given by,

$$\boxed{F_m = \frac{F}{K}}$$

Q6 ⇒ What is meant by the "test charge" and "Source charge"?

Ans ⇒ The charge on which the force is exerted by the another charge is called test charge. Then, while the charge which exerts the force on the test charge is called source charge.

Q7. While the definition of intensity of Electric field, the test charge should be infinitishamly small. Expl

or

What is the advantage of defining

$$\vec{E} = \lim_{q_0 \to 0} \left[ \frac{\vec{F}}{q_0} \right]$$

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Q8. Why two Electric lines of force do not cross each other?

Ans⇒ This is because, the Intensity of Electric field at any point has a Single value. But, If the two Electric lines of force (intersect) cross each other Then, there will be two tangents hence two values of the Electric field at that point, which is not possible.

Q9. What do you understand by the point charge in physics?

Ans⇒ By point charge, we mean on the charged body whose Spatial dimensions are very - 2 small as compared to the other distances involved the problem under consideration. Then, The point charge does not influence the Electric field produced by the source charge.

Q10. Define Electric field. What are its units?

Ans⇒ Electric field: The space around the charged body with which its electric influence (force of attraction and repulsion) can be explaine by the other charge is called Electric field.

Units of Electric field.

SI : ($NC^{-1}$).

CGS: (dyne/stat Coulomb).

Q11. Coulomb's Force is two body interaction." Explain.

Ans => The Coulomb's Force betn. the two charged bodies is not affected by the presence of another charged bodies, hence, Coulomb's force is two body interaction.

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Q12. What are the limitations of Coulomb's law?

Ans => (i) This law holds good for the charge at rest.
(ii). This law holds good for point charge because, If the charges are Extended, then it is difficult to determine the distance betn. them.

Q13. Is it possible to have finite Electric field due to an ~~Electric~~ Infinite charge distribution? If so, give an Example?

Ans => Yes, The value of Electric field due to an infinite line of charge is finite

$$\left[E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}\right].$$

Q14. Is the volume charge density is invariant?

Ans => Volume charge density $= \frac{\text{Charge}}{\text{Volume}}$, since, the volume is not invariant, the volume charge density is also not then the volume charge density is also not invariant.

Q15. What do you understand by Electric flux?

Ans ⇒ Electric flux through any surface area is defined as the no. of electric lines of force ~~pass~~ through the given area is called Electric flux

Q16. State Gauss theorem.

Ans ⇒ According to this theorem, the total Electric Flux through a closed surface is Equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the surface. i.e $\phi = \oint\!\!\oint_S \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0}$.

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Q17. Write down Gauss's law in differential form

Ans ⇒ $\operatorname{div}\vec{E} = \rho/\varepsilon_0$. ⇒ $\rho$ is the volume charge density.

Q18. What do you understand by the Electric flux density?

Ans ⇒ Electric flux density is defined as the Electric f per unit area.

Q19. What do you understand by the Gaussian surf

Ans ⇒ An imaginary closed surface Containing net electric charge is called Gaussian Surface. The Electric field intensity at Every point on th

Gaussian surface is constt.

Q20. What is the Electric flux through a closed surface surrounding an Electric dipole?

Ans ⇒ Electric dipole consists of two equal and opposite charges. Hence, the net charge inside the closed surface is zero. Electric flux through a closed surface surrounding an Electric dipole is zero.

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Q21. A charged conductor has Energy. Explain.

Ans ⇒ A charged conductor is always under a stress acting outward due to the electric field. Due to this, surface of the conductor is displaced and work is done. This work done is stored as the Energy of the conductor.

Q22. Derive units of Electric flux.

Ans ⇒ Electric flux, $d\phi = \vec{E}.\vec{ds}$.

So, unit of Electric flux = unit of $\vec{E}$ × unit of $\vec{ds}$

$= NC^{-1} \times m^2$

$= Nm^2C^{-1}$.

Units of Electric flux $= Nm^2C^{-1}$

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# CHAPTER-III
# Electrostatic Potential

## • Electrostatic potential Energy :—

Electrostatic potential Energy of a charge at a point in the Electric Field due to any charge $\vec{E}$ (called source charge) work done by an External Force (beginning) bringing a test charge from infinity to that point in the Electric Field.

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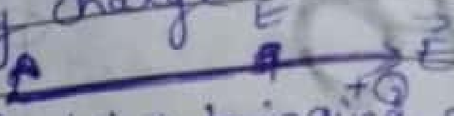


Let a +ve test charge $q_0$ is placed in the Electric field due to another positive charge.

The force acting on the test charge in the Electric field $\vec{E}$ is,

$$\Rightarrow \vec{F} = q_0 \vec{E}.$$

The force moves to test charge in the directn. of Electric Field.

Suppose $\vec{F_0}$ is an External Force to overcome the force $\vec{F_e}$ on the test charge move it without any acceleration towards Source Charge.

If the test charge is displaced by $d\vec{r}$ from point A to point B in the Electric field.
Work done by the External force is,

$$W_{AB} = \int_A^B \vec{F_0} \cdot d\vec{l}.$$

$$\vec{F_0} = -\vec{F_e} = -q_0\vec{E}$$

$$\Rightarrow W_{AB} = -\int_A^B q_0\vec{E}.\vec{dl} = -q_0\int_A^B \vec{E}.\vec{dl}.$$

78 This work done gives the difference in Electrical potential Energy of the charge field Systm betn. points A and B.

$$\boxed{W_{AB} = U_B - U_A = -q_0\int_A^B \vec{E}.\vec{dl}.}$$

⇒ If point A is at infinity, then there is no Electrostatic force of (attraction) betn. source charge Q and test charge $q_0$ at infinity is zero. The work done to displaces to test charge from infinity to the point B in the Electric field.

$$U_B - U_\infty = U_{\infty B} = U_B - 0 = W_B.$$

$$\Rightarrow U_B = W_{\infty B} = -q_0\int_\infty^B \vec{E}.\vec{dl}.$$

- This work done is Equal to the Electrostatic potential Energy of the System at any point in the Electric field.

- Electrostatic potential and potential difference :-

⇒ It may be defined as that the work done per unit positive charge from infinity to that point against Electrostatic Force of electric field of the source charge called potential difference.

$V = \frac{U}{q_0}$

Let $U$ be the Electrostatic potential Energy at a point in the Electric Field. Electric potential at that point of the Electric Field is,

$$\Rightarrow V = \frac{U}{q_0}.$$

potential Energy on a test charge $q_0$ at a point in the Electric field,

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$$\Rightarrow U = -q_0 \int_{\infty}^{B} \vec{E} \cdot d\vec{r}.$$

Electric potential at that point in the Electric field is,

$$V = \frac{U}{q_0} = -\int_{\infty}^{B} \vec{E} \cdot d\vec{r}.$$

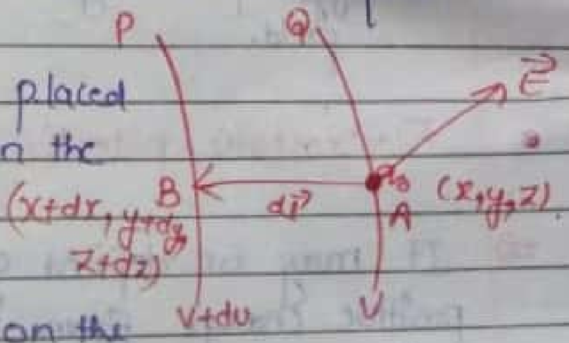
⇒ Electric potential is a Scalar quantity.

- SI unit of Electric potential is Volt (V).

## ⇒ Relation Betn Electric field ($\vec{E}$) and Electric potential :-

Consider two Equipotential Surfaces P and Q having Electric potential V and dV an Electric field.

Let a test charge $q_0$ is placed at a point A $(x, y, z)$ on the Equipotential Surface Q.




The force acting on the test charge in the Electric field is,

$$\vec{F} = q_0\vec{E} \quad \text{---(i)}$$

Let the test charge $q_0$ is moved from point $A(x,y,z)$ on equipotential surface Q to point $B(x+dx, y+dy, z+dz)$ on the equipotential surface P by an external force surface without any acceleration.

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External force acting on the test charge:-

$$\vec{F_e} = -\vec{F} = -q_0\vec{E}$$

⇒ If $d\vec{r}$ is the displacement of the test charge from point A to point B, work done of charge is,

$$dw = \vec{F_e}\cdot d\vec{r} = -q_0\vec{E}\cdot d\vec{r}$$

$$\frac{dw}{q_0} = \vec{E}\cdot d\vec{r}$$

$$\Rightarrow \frac{dw}{q_0} = dv$$

$$\Rightarrow dv = -\vec{E}\cdot d\vec{r}$$

$$\Rightarrow dv = \frac{dv}{dx}dx + \frac{dv}{dy}dy + \frac{dv}{dz}dz$$

$$= \left[\frac{d}{dx}v\hat{i} + \frac{d}{dy}v\hat{j} + \frac{d}{dz}v\hat{k}\right]\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

(ii)

$$= \left[\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right]v\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\boxed{dv = \vec{\nabla}v\cdot d\vec{r}}$$

$$\Rightarrow \vec{\nabla}v\, d\vec{r} = -\vec{E}\, d\vec{r}$$

$$\Rightarrow \boxed{\vec{E} = -\vec{\nabla}v}$$

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$\boxed{\vec{E} = -\vec{\nabla} V}$ Ans.

⇒ Electric dipole :- A System of two Equal and opposite charges separated by certain $dist^n$. Called Electric dipole.

⇒ Electric Dipole Moment :- ($\vec{p}$) The product of the magnitude of Either charge ~~dipole moment~~ is of the Electric dipole at the dipole length are Called Electric Dipole Moment.

$$\boxed{\vec{P} = q \times 2\vec{l}}$$

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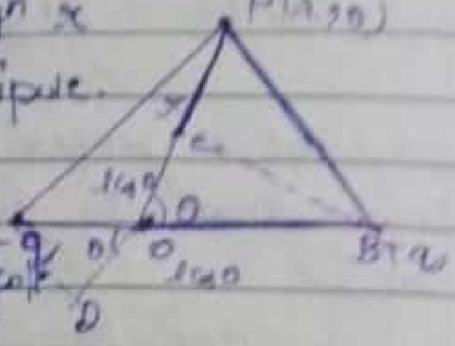
⇒ SI unit of Electric Dipole Moment is Coulomb meter (cm).

Physics:-

# * Electric Potential Due to an Electric Dipole



Consider an Electric dipole. Let us Calculate the potential V at point P at distⁿ $x$ from mid point O of the dipole.




Date 19/04/20 The line joining P and OA makes an angle θ with the dipole moment $\vec{p}$

Co-ordinates of point $P(x, \theta)$

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Potential at P due to +q charge is :-

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{BP}$$

Potential at P due to -q charge is :-

$$V = \frac{1}{4\pi\epsilon_0} \frac{-q}{AP}$$

Total potential at P due to the dipole is :-

$$V = V_1 + V_2 = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{BP} - \frac{1}{AP}\right] \quad \text{--- (1)}$$

$BP = PC = OP - OC = x - l\cos\theta$

$AP = DC = OP - OD = x + l\cos\theta$

Put these Values in Eqn. (1).

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(x - l\cos\theta)} - \frac{1}{(x + l\cos\theta)}\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{(x + l\cos\theta) - (x - l\cos\theta)}{x^2 - l^2\cos^2\theta}\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{\cancel{x} + l\cos\theta - \cancel{x} + l\cos\theta}{x^2 - l^2\cos^2\theta}\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{2l\cos\theta}{x^2 - l^2\cos^2\theta}\right]$$

i.e $q \times 2l = P$ (dipole moment of Electric dipole).

$$V = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{(x^2 - l^2\cos^2\theta)} \quad \text{--- (2)}$$

If point P is far away from the centre of the electric dipole, then $x^2$ is large as compared to $l^2\cos^2\theta$

i.e. Then $l^2\cos^2\theta$ can be neglected

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i.e. $$V = \frac{p\cos\theta}{4\pi\varepsilon_0 x^2} \quad \text{--- (3)}$$

$\theta$ is the angle betn $\vec{P}$ and $\vec{x}$

$$\boxed{V = \frac{\vec{p}\cdot\hat{x}}{4\pi\varepsilon_0 x^2}} \quad \text{--- (4)}$$ where $\hat{x}$ is unit vector.

$$\boxed{\Rightarrow \vec{\nabla}\left[\frac{1}{x}\right] = -\frac{\hat{x}}{x^2}}$$

$\vec{\nabla}\left[\frac{1}{x}\right] = \frac{\hat{x}}{x^2}$

Now, Eqn. (4) becomes,

$$V = -\frac{1}{4\pi\varepsilon_0} \vec{p}\cdot\vec{\nabla}\left(\frac{1}{x}\right) \quad \text{--- (5)}$$

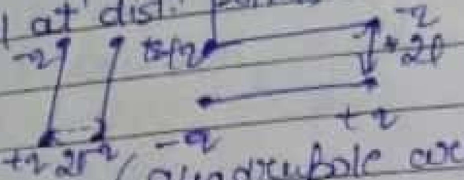
(93/100)

- Electric potential due to an Electric dipole is inversely proportional to the square of the dist of the point of observation from the Electric dipole. Then, $V \propto \frac{1}{x^2}$
  Electric potential due to Electric dipole decreases rapidly with dist$^n$ $x$.

If $V = 0$ at all points at which $\theta = 90°$.
Electric potential due to an electric dipole at all points on the $\perp$lar bisector of the Electric dipole is zero.
It means that no work is done bring a test charge from infinity to any point on the $\perp$lar bisector of the Electric dipole.

## Electric Quadrupole:—

* A quadrupole consists of two equal and opposite charge dipole seperated by a small dist$^n$. They do not coincide in space so that their electric effects do not cancel at dist$^n$ points.




(quadrupole arrangements)

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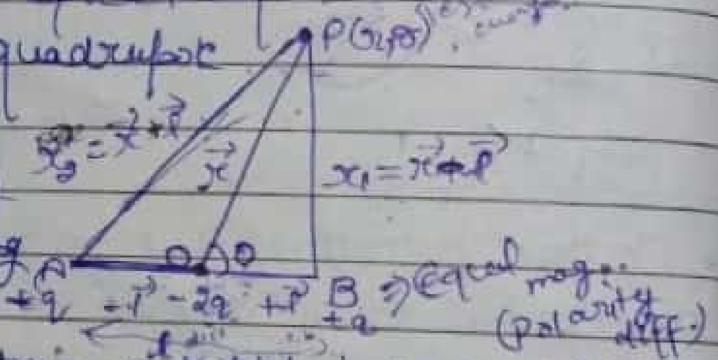
Quadrupole moment is:—

$$Q_d = 2ql^2.$$

$\Rightarrow$ Unit of Quadrupole is $Cm^2$. Ans

## Potential Due to a Quadrupole:—

Consider a linear quadrupole




Let P be any point having position vector P $(x, \theta)$ ~~having~~ $x$ from charge $-2q$ of the quad. Electric potential is to be calculated.

Let $\vec{OP} = \vec{x}$, $\vec{BP} = \vec{x}_1$ and $\vec{AP} = \vec{x}_2$.

Using $\Delta$ law of vector add$^n$:—

$\vec{OA} + \vec{AP} = \vec{OP}$ or $\vec{AP} = \vec{OP} - \vec{OA}$

$\vec{AP} = \vec{x} - (-\vec{l})$

$\vec{AP} = \vec{x} + \vec{l}$.

$\vec{OB} + \vec{BP} = \vec{x}$
$\vec{BP} = \vec{x} - \vec{OB}$
$\vec{BP} = \vec{OP} - \vec{OB}$
$= \vec{x} + \vec{l}$

i.e. lly, $\vec{BP} = \vec{x} + \vec{l}$

Potential at P due to +q charge is given by:—

$$\boxed{V_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{x_1}}$$

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{-2q}{x}$$

Potential at p due to +q at B is:—

$$V_3 = \frac{1}{4\pi\epsilon_0}\frac{q}{x_2}$$

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Total Potential at p due to quad.

$$V = V_1 + V_2 + V_3$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{x_1} - \frac{2q}{x} + \frac{q}{x_2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{x_1} - \frac{2}{x} + \frac{1}{x_2}\right]$$

Using Δ law

i.e. $x_1 = |\vec{x} + \vec{l}| = (x^2 + l^2 - 2xl\cos\theta)^{1/2}$

θ is the angle betn $\vec{x}$ and $\vec{l}$.

$$x_1 = (x^2 + l^2 - 2xl\cos\theta)^{1/2}.$$

$$\Rightarrow \frac{1}{x_1} = \frac{1}{(x^2 + l^2 - 2xl\cos\theta)^{1/2}}$$

$$\frac{1}{x_1} = \frac{1}{x\left[1 + \frac{l^2}{x^2} - \frac{2l}{x}\cos\theta\right]^{1/2}} \Rightarrow \frac{1}{x_1} = \frac{1}{x}\left[1 + \left(\frac{l^2}{x^2} - \frac{2l}{x}\cos\theta\right)\right]^{-1/2}$$

$$\boxed{\frac{1}{x_1} = \frac{1}{x}\left[1 + \left(\frac{l^2}{x^2} - \frac{2l}{x}\cos\theta\right)\right]^{-1/2}}$$

$\cos\theta = \frac{BC}{B}$

$AD = \sqrt{(a + b\cos\theta)^2 + (b\sin\theta)^2}$

$= \sqrt{a^2 + b^2\cos^2\theta + 2ab\cos\theta + b^2\sin^2\theta}$

θ is ∠ betn vectors tail to tails

Using Binomial theo. $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$

$$\frac{1}{x_1} = \frac{1}{x}\left[1 - \frac{1}{2}\left(\frac{l^2}{x^2} - \frac{2l}{x}\cos\theta\right) + \frac{(-1/2)(-1/2-1)}{2}\left(\frac{l^2}{x^2} - \frac{2l}{x}\cos\theta\right)^2 + \right.$$

$$\Rightarrow \frac{1}{x_1} = \frac{1}{x}\left[1 - \frac{1}{2}\frac{l^2}{x^2} + \frac{2l}{2x}\cos\theta + \frac{(-1/2)(-3/2)}{2}\left(\frac{l^4}{x^4} - \frac{4l^2\cos^2\theta}{x^2}\right) + \right.$$

Since $x >> l$, i.e. $\frac{l}{x^2}$ and neglecting higher terms:—

$$\Rightarrow \frac{1}{x_1} = \frac{1}{x}\left[1 - \frac{1}{2}\frac{l^2}{x^2} + \frac{l}{x}\cos\theta + \frac{3}{8} \times \frac{4l^2}{x^2}\cos^2\theta\right].$$

$$\Rightarrow \frac{1}{x_1} = \frac{1}{x}\left[1 - \frac{1}{2}\frac{l^2}{x^2} + \frac{l}{x}\cos\theta + \frac{3}{2}\frac{l^2}{x^2}\cos^2\theta\right].$$

$\frac{l^2}{x^2}$ can be neglected

$$\frac{1}{x_1} = \frac{1}{x}\left[1 + \frac{l^2}{2x^2}(3\cos^2\theta - 1) + \frac{l}{x}\cos\theta\right].$$

$\frac{1}{x_2} = \frac{1}{x}\left[1+\frac{l^2}{2x^2}(3\cos^2\theta-1)-\frac{l}{x}\cos\theta\right].$

substitute the value of $\frac{1}{x_1}$ and $\frac{1}{x_2}$ in Eqn (i).

Now, $V = \frac{q}{4\pi\epsilon_0 x}\left[1+\frac{l^2}{2x^2}(3\cos^2\theta-1)+\frac{l}{x}\cos\theta\right\} - 2 +$

$1+\frac{l^2}{2x^2}(3\cos^2\theta-1)-\frac{l}{x}\cos\theta\Big].$

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$V = \frac{q}{4\pi\epsilon_0 x}\left[\frac{l^2}{\cancel{x^2}}(3\cos^2\theta-1)\right].$

$\Rightarrow V = \frac{ql^2}{4\pi\epsilon_0 x^3}\left[3\cos^2\theta-1\right].$

Dividing and multiplying by (2) ~~Eqn~~

$\Rightarrow V = \frac{2ql^2(3\cos^2\theta-1)}{8\pi\epsilon_0 x^3}$

$2ql^2 = Q_d$ Quad. moment

$= \frac{Q_d(3\cos^2\theta-1)}{8\pi\epsilon_0 x^3} \Rightarrow V = \frac{Q_d(3\cos^2\theta-1)}{8\pi\epsilon_0 x^3}.$

Potential due to linear quad. varies inversely proportional to cube of the distn.

Spec. Cases:-

If point P on the axial line of quad. $\theta = 0°$.

$V = \frac{Q_d(3-1)}{8\pi\epsilon_0 x^3} \Rightarrow V = \frac{Q_d}{4\pi\epsilon_0 x^3}.$

If point P on the Equatorial line of quad.

$\theta = 90°$.

$\Rightarrow \boxed{V = \frac{Q_d}{8\pi\epsilon_0 x^3}}$ Ans.

900,000 1900 1000 739 759 3071 (1) 990,000,046 82595

$\Rightarrow V = \frac{-1}{4\pi\varepsilon_0} \vec{P} \cdot \vec{\nabla}\left(\frac{1}{x}\right)$, Electric potential due to Electric dipole is inversely proportional to the square of the dist$^n$ of the point of observation.

$\Rightarrow$ From V = 0 at all points at which $\theta = 90°$. Electric potential due to electric dipole at all points on the perpendicular bisector of the Electric dipole is zero. No work done to bring a test charge from infinity to any point on the $\perp$ bisector of the Electric field.

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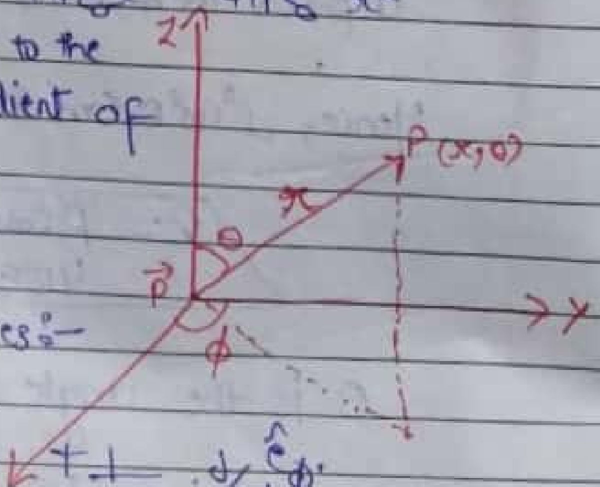
## Electric field of a Dipole :-

The Electric potential due to an Electric dipole of dipole moment $\vec{P}$ at point $P(x, \theta)$.

$$V(x,\theta) = \frac{P\cos\theta}{4\pi\varepsilon_0 x^2} = \frac{1}{4\pi\varepsilon_0} \frac{\vec{P}\cdot\hat{x}}{x^2}$$

- Electric field is Equal to the negative gradient of potential.

$$\vec{E} = -\vec{\nabla}V.$$




In Spherical Co-ordinates:-

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{e}_r + \frac{1}{x}\frac{\partial}{\partial\theta}\hat{e}_\theta + \frac{1}{x\sin\theta}\frac{\partial}{\partial\phi}\hat{e}_\phi.$$

where, $\hat{e}_r$, $\hat{e}_\theta$ and $\hat{e}_\phi$ are unit vectors in the direct$^n$ inc. $x$, $\theta$ and $\phi$.

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{e}_x + \frac{1}{x}\frac{\partial}{\partial \theta}\hat{e}_\theta + \frac{1}{x\sin\theta}\frac{\partial V}{\partial \phi}\hat{e}_\phi\right].$$

$$\boxed{\vec{E} = E_x\hat{e}_x + E_\theta\hat{e}_\theta + E_\phi\hat{e}_\phi}$$

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$E_x$, $E_\theta$ and $E_\phi$ are the unit vectors components of Electric field $\vec{E}$ in the directn of $\hat{e}_x$, $\hat{e}_\theta$ and $\hat{e}_\phi$.

$$\Rightarrow E_x\hat{e}_x + E_\theta\hat{e}_\theta + E_\phi\hat{e}_\phi = \left[-\frac{\partial V}{\partial x}\hat{e}_x - \frac{1}{x}\frac{\partial}{\partial \theta}\hat{e}_\theta - \frac{1}{x\sin\theta}\frac{\partial V}{\partial \phi}\hat{e}_\phi\right]$$

Compairing the coeffi. of $\hat{e}_x$, $\hat{e}_\theta$ and $\hat{e}_\phi$.

$$E_x = -\frac{dV}{dx}$$

$$E_\theta = -\frac{1}{x}\frac{\partial V}{\partial \theta}$$

$$E_\phi = \frac{-1}{x\sin\theta}\frac{\partial V}{\partial \phi}.$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{p\cos\theta}{4\pi\varepsilon_0 x^2}\right]$$

$$= -\frac{p\cos\theta}{4\pi\varepsilon_0}\frac{\partial}{\partial x}(x^{-2}).$$

$$= \frac{2p\cos\theta}{4\pi\varepsilon_0 x^3}$$

$$E_\theta = \frac{-1}{x}\frac{\partial}{\partial \theta}\left[\frac{p\cos\theta}{4\pi\varepsilon_0 x^2}\right] = \frac{-p}{4\pi\varepsilon_0 x^3}\frac{\partial}{\partial \theta}(\cos\theta).$$

$$= \frac{-p}{4\pi\varepsilon_0 x^3}(-\sin\theta) = \frac{p\sin\theta}{4\pi\varepsilon_0 x^3}.$$

$= \frac{p\sin\theta}{4\pi\varepsilon_0 x^3}$

$$E_\phi = \frac{1}{x\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{p\cos\theta}{4\pi\varepsilon_0 x}\right) = \frac{-1}{x\sin\theta} \times 0 = 0.$$

Electric field of a dipole at point p is given by,

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$$\vec{E}(x,\theta) = E_x\hat{e}_x + E_\theta\hat{e}_\theta + E_\phi\hat{e}_\phi.$$

$$= \frac{2p\cos\theta}{4\pi\varepsilon_0 x^3}\hat{e}_r + \frac{p\sin\theta}{4\pi\varepsilon_0 x^3}\hat{e}_\theta + 0.$$

$$\vec{E}(x,\theta) = \frac{1}{4\pi\varepsilon_0}\frac{p}{x^3}(2\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta).$$

Magnitude of Electric ~~dipole~~ field of a dipole is,

$$|\vec{E}(x,\theta)| = \frac{1}{4\pi\varepsilon_0}\frac{p}{x^3}\sqrt{4\cos^2\theta + \sin^2\theta}.$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{x^3}\sqrt{3\cos^2\theta + 1}.$$

$$\boxed{|\vec{E}(x,\theta)| = \frac{1}{4\pi\varepsilon_0}\frac{p}{x^3}\sqrt{3\cos^2\theta + 1}.}$$

Electric field due to quadrupole:-

Electric field at point p due to quadrupole has two components (i) Radial component (ii) Transverse component

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(i) Radial component of electric field is, $(E_r)$

$$E_r = -\frac{dV}{dr} = -\frac{d}{dr}\left[\frac{1}{4\pi\varepsilon_0}\frac{Qa}{2r^3}(3\cos^2\theta - 1)\right]$$

$$\Rightarrow E_r = -\frac{Qa}{4\pi\varepsilon_0}\frac{(3\cos^2\theta-1)}{2}\frac{d}{dr}(r^{-3})$$

$$= -\frac{Qa}{4\pi\varepsilon_0}\frac{(3\cos^2\theta-1)}{2}(-3)r^{-4}$$

$$E_r = \frac{1}{4\pi\varepsilon_0}\frac{3Qa(3\cos^2\theta-1)}{2r^4}$$

(ii) Transverse component :-

$E_\theta$

$$E_\theta = -\frac{1}{r}\frac{dV}{d\theta} = -\frac{1}{r}\frac{d}{d\theta}\left[\frac{1}{4\pi\varepsilon_0}\frac{Qa}{2r^3}(3\cos^2\theta - 1)\right]$$

$$= -\frac{1}{r}\times\frac{1}{4\pi\varepsilon_0}\frac{Qa}{2r^3}\frac{d}{d\theta}(3\cos^2\theta-1)$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{Qa}{2r^4}\left[3\times 2\cos\theta(-\sin\theta)\right]$$

$$E_\theta = \frac{1}{4\pi\varepsilon_0}\frac{3Qa\,2\cos\theta\sin\theta}{2r^4}$$

$$E_\theta = \frac{1}{4\pi\varepsilon_0}\frac{3Qa\,2\sin2\theta}{2r^4}$$

$$\boxed{E_\theta = \frac{1}{4\pi\varepsilon_0}\frac{3Qa\sin2\theta}{2r^4}}$$

Resultant Electric field at point P due to quadrupole :—

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{2x^4}\left[\sqrt{(3\cos^2\theta - 1)^2 + (\sin 2\theta)^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{2x^4}\left[\sqrt{9\cos^4\theta + 1 - 6\cos^2\theta + (2\sin\theta\cos\theta)^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{2x^4}\left[9\cos^4\theta + 1 - 6\cos^2\theta + (2\sin\theta\cos\theta)^2\right]^{1/2}$$

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$$= \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{2x^4}\left[9\cos^4\theta + 1 - 6\cos^2\theta + 4\sin^2\theta\cos^2\theta\right]^{1/2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{2x^4}\left[9\cos^4\theta + 1 - 6\cos^2\theta + 4(1 - \cos^2\theta)\cos^2\theta\right]^{1/2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{2x^4}\left[9\cos^4\theta + 1 - 6\cos^2\theta + 4\cos^2\theta - 4\cos^4\theta\right]^{1/2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{2x^4}\left[5\cos^4\theta - 2\cos^2\theta + 1\right]^{1/2}.$$

- Special Cases:—

If $\theta = 0^\circ$ it lies at point on the axial line

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{2x^4}\left[5\cos^4(0) - 2\cos^2(0) + 1\right]^{1/2}.$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{2x^4}\left(5 - 2 + 1\right)^{1/2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{2x^4}\left[3 + 1\right]^{1/2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{2x^4}\left[4\right]^{1/2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{3Qd \times 2}{2x^4}$$

$$\boxed{E_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{3Qd}{x^4}}$$

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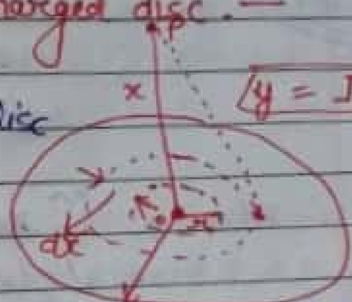
(iii) If $\theta = 90^\circ$, at point p lies on the equitorial line of the quadrupole

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{3Qa}{2r^4}\left[5\cos^4(90^\circ) - 2\cos^2(90^\circ)+1\right]^{1/2}.$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{3Qa}{2r^4}\left[5(0) - 2(0)+1\right]^{1/2}.$$

$$\Rightarrow \boxed{E_{equ} = \frac{1}{4\pi\varepsilon_0}\frac{3Qa}{2r^4}} \Rightarrow \boxed{E_{axial} = 2 \times E_{equ}}$$

- ★ Electric potential Due to charged disc :—



Consider a uniformly charged disc at dist$^n$. R having surface charge density $\sigma$.

$\boxed{y = \sqrt{x^2+x^2}}$

Let q be the charged disc is on the at point p. Point p is on the axis of disc at dist$^n$. x.

Now, The disc is assumed to be concentric rings ~~0 onto~~ R.

The area of the ring = $2\pi x\,dx$.

$\Rightarrow$ charge on the ring $dq = \sigma \times 2\pi x\,dx$ i.e $\sigma = \frac{q}{4\pi R^2}$

Electric potential at p due to ring ~~charged disc~~.

$$\int dv = \frac{q}{4\pi\varepsilon_0}\frac{dq}{(r^2+x^2)^{1/2}} = \frac{q}{4\pi\varepsilon_0}\frac{\sigma \times 2\pi x\,dx}{(r^2+x^2)^{1/2}}$$

Electric potential due to charged disc is,

$$\int dv = \int_0^R \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\pi x\sigma dx}{(x^2+x^2)^{1/2}}$$

$$V = \frac{\sigma}{2\varepsilon_0} \int_0^R \frac{x\,dx}{(x^2+x^2)^{1/2}} \quad \text{--- (2)}$$

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$\Rightarrow x = \sqrt{y^2+x^2}$

$\Rightarrow \boxed{x^2+x^2 = y}$

$y = x^2+x^2$

$2x\,dx = dy$

$\Rightarrow \boxed{x\,dx = \frac{dy}{2}}$

Now, $\int_0^R \frac{x\,dx}{(x^2+x^2)^{1/2}} = \int_{x^2}^{x^2+R^2} \frac{(y)^{-1/2}dy}{2} = \left[y^{1/2}\right]_{x^2}^{x^2+R^2}$

$$\int_0^R \frac{x\,dx}{(x^2+x^2)^{1/2}} = \left[(x^2+R^2)^{1/2} - x\right] \quad \text{--- (3)}$$

Now, using Eqn. (2) & (3) we get.

$$V = \frac{\sigma}{2\varepsilon_0}\left[(x^2+R^2)^{1/2} - x\right] \quad \text{--- (4)}$$

Which is the required expression for the electric potential due to charged ~~dissing~~ at dist$^n$ x on the axis of the disc.

$$\Rightarrow (x^2+R^2)^{1/2} = x\left[1+\frac{R^2}{x^2}\right]^{1/2}$$

$$= x\left[1+\frac{1}{2}\cdot\frac{R^2}{x^2}+ \quad\right]$$

Hence Eqn. (4) can be written as,

$$V = \frac{\sigma}{2\varepsilon_0}\left[\cancel{x} + \frac{R^2}{2x} - \cancel{x}\right] \Rightarrow V = \frac{\sigma}{2\varepsilon_0}\left[\frac{R^2}{2x}\right].$$

$$\Rightarrow V = \frac{\sigma R^2}{4\varepsilon_0 x}.$$

i.e $\sigma = \frac{q}{4\pi R^2}$

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$$\Rightarrow V = \frac{\frac{q}{4\pi \cancel{R^2}} \times \cancel{R^2}}{\frac{4\varepsilon_0 x}{1}} \Rightarrow V = \frac{q}{4\pi} \times \frac{1}{4\varepsilon_0 x}$$

$$\Rightarrow \boxed{V = \frac{q}{16\pi\varepsilon_0 x}}$$

$\Rightarrow \boxed{V = \frac{1}{16\pi\varepsilon_0}\cdot\frac{q}{x}}$ i.e which is Equal to the Electric potential due to point charge at dist$^n$ $x$. Ans

Q1:- An Electrostatic field is curl free? Explain why?

Ans:- Acc. to Electric field is -ve gradient of potential

$\vec{E} = -\vec{\nabla}V$.

Taking curl on both sides,

$\Rightarrow \vec{\nabla}\times\vec{E} = -\vec{\nabla}\times\vec{\nabla}V$.

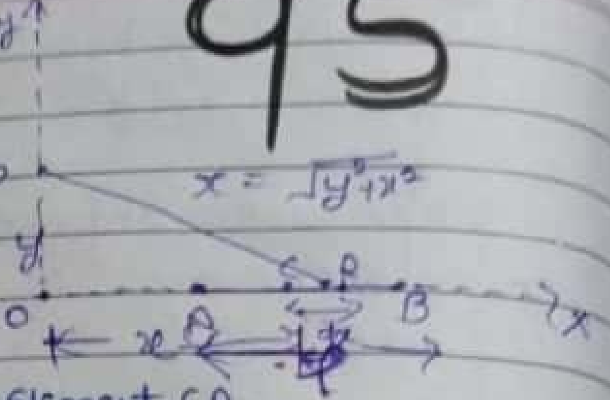
$\Rightarrow$ But $-\vec{\nabla}\times\vec{\nabla}V$ i.e Curl of grad. of Scalar function is zero.

$\Rightarrow \vec{\nabla}\times\vec{E} = 0$.

i.e An Electrostatic Field is curl free or Non-Curl field. Ans

## ★ Electric potential due to a long uniformly charged wire :-

Consider a long uniformly charged wire AB of length and linear charge density $\lambda$. Let $q$ be the charge uniformly distributed on the wire.




The charge on the element CD,

$$dq = \underbrace{\lambda}_{\text{surface charge density}} dx = \frac{q}{l}dx \; ;$$

Electric potential due to small element at a point P, dist$^n$ $r$ from the charged element is:-

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{l}\frac{dx}{r}$$

Electric potential at point P due to whole wire is

$$\int dV = \int_a^{l+a} \frac{1}{4\pi\varepsilon_0}\frac{q\,dx}{lr} = \int_a^{l+a} \frac{1}{4\pi\varepsilon_0}\frac{q\,dx}{l(y^2+x^2)^{1/2}}$$

$$V = \frac{q}{4\pi\varepsilon_0 l}\int_a^{l+a} \frac{dx}{(y^2+x^2)^{1/2}}$$

$= \frac{1}{(x^2+y^2)^{1/2}}$

log(

$$V = \frac{q}{4\pi\varepsilon_0 l}\left[\log_e\left\{x + \sqrt{y^2+x^2}\right\}\right]_a^{l+a}$$

$$= \frac{q}{4\pi\varepsilon_0 l}\left[\log_e\left\{(l+a) + \sqrt{y^2+(l+a)^2}\right\} - \log_e\left\{a + \sqrt{y^2+a^2}\right\}\right]$$

$$\Rightarrow \boxed{V = \frac{q}{4\pi\varepsilon_0 l}\left[\log_e \frac{[(l+a) + \sqrt{y^2+(l+a)^2}]}{[a+\sqrt{y^2+a^2}]}\right]} \quad (1)$$

Sp. cases:—

(i) If point P lies on x-axis then y = 0.

Eqn. ① becomes:—

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{l}\left[\log_e\left\{\frac{2(l+a)}{2a}\right\}\right].$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{l}\left[\log\left(1+\frac{l}{a}\right)\right].$$

If $l << a$, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$, which is the value of pot^n due to point charge q.

(ii) If point P is ⊥ar to the length of the wire. Then, a = 0, Eqn. ① becomes

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{l}\left[\log\left[\frac{l+\sqrt{y^2+l^2}}{a+\sqrt{y^2+a^2}}\right]\right].$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{l}\left[\log\left[\frac{l}{y}+\sqrt{1+\frac{l^2}{y^2}}\right]\right].$$

If Point P is far away from the wire $l << y$,

i.e. $\left[1+\frac{l^2}{y^2}\right]^{1/2} = 1+\frac{l^2}{2y^2}+\cdots$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{l}\left[\log_e\left(1+\frac{l}{y}\right)\right] = \frac{1}{4\pi\varepsilon_0}\frac{q}{l}\frac{l}{y}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{y}}$$ i.e. long uniformly charged wire behave as a point charge if the dist^n of the point of obs. is far away from the wire.

(Electrostatic Potential)

① Explain the terms (i) Electric potential
(ii) Electric potential difference.

Ans ⇒ Electric potential: The amount of work done to bring a unit +ve charge from infinity to a given point in the electric field is called potential at that point is called

97 Electric potential.

Electric potential difference: The amount of work done to move a unit +ve charge from one point to another point in the Elec field is called Electric potential difference.

Q2. What is the relation bet^n Electric field and Electric potential?

Ans ⇒ $\vec{E} = -\vec{\nabla} V$, $\vec{E}$ is Electric field and V is Electric potential.

Q3. What is the significance of -ve sign in the Equation.

$$\vec{E} = -\vec{\nabla} V$$

Ans ⇒ Negative sign shows that the electric field p in the direc^n of decreasing potential.

Q4. What will be the value of $\vec{E}$ (i) in a region where potential V is constant & (ii) at a point where $V = 0$?

Ans ⇒ We know,

$$\boxed{\vec{E} = -\vec{\nabla} V}$$

(i) when $V =$ constant, $\vec{E} = 0$.
(ii) when $\vec{E} =$ const., then $V = 0$.

Q5. An Electrostatic field is curl free. Explain why?

Ans ⇒ $\vec{E} = -\vec{\nabla} V$.

- Taking curl on both sides,

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$$\vec{\nabla} \times \vec{E} = -\vec{\nabla} \times \vec{\nabla} V.$$

i.e. $\vec{\nabla} \times \vec{\nabla} V$, curl of gradient of scalar function is zero.

$$\boxed{\vec{\nabla} \times \vec{E} = 0}$$

Thus, Electrostatic field $\vec{E}$ is curl free or non-curl field

# CHAPTER- 4

## Electric current and Invariance of charge.

**Electric Current:—** The Electric Current in a Conductor depends ~~upon~~ on the flow of the amount of Electric charge (negative) in the conductor.

- Amount of electric charge flowing through cross-section of cond. per unit time.

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Let $Q$ be the Electric charge crossing through a cross-section of a conductor in time $t$, then Electric current ($I$) through the Conductor is given by

$$I = \frac{Q}{t}$$, In $n$ Electrons each charge crossing through at cross-section of conductor in time $t$, then the total Electric charge passing through a cross-section of the conductor is

$$Q = ne, \quad I = \frac{Q}{t} = \frac{ne}{t}.$$

If $\nabla Q$ be the net charge crossing a cross-section of conductor in time $\Delta t$, instantaneous value of Electric current through the conductor is

$$\boxed{I(t) = \lim_{\nabla t \to 0} \left[\frac{\Delta Q}{\Delta t}\right] = \frac{dQ}{dt}}$$

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**Unit of current:** (i) SI Syst. The unit of current is Ampere (A).

Current is Equal to the one Ampere if one Coulomb of charge flows through the cross-section of the conductor in one sec.

$$1A = \frac{1C}{1S} \quad CS^{-1} \Rightarrow 1C = 3\times10^{9}\, e.s.u$$

$$1A = \frac{1C}{1s} = \frac{3\times10^{9}\, \text{Stat-Coulomb}}{1s} = 3\times10^{9}\ \text{Stat ampere}$$

(ii) C.G.S unit:— Current is equal to 1 e.m.u, if 1 e.m.u of charge ($3\times10^{10}$ esu of charge $= 10C$) flows through the cross-section of the conductor in one second.

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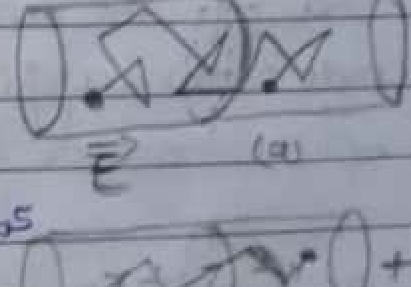
$$1\, e.m.u = \frac{1\, e.m.u \text{ of charge}}{1\, s}$$

$$= \frac{3\times10^{10}\, e.s.u \text{ of charge}}{1s}$$

$$= 3\times10^{10}\, e.s.u \text{ of current}$$

## ☆ Electric Current in Conductors:—



$\vec{E}$ (a) (b)

In a conductor, the free Electrons move randomly in Zig-Zag path with a thermal speed of order of $10^5$ to $10^6\ ms^{-1}$ at room temp. The no. of Electrons (-ve charges) crossing a given cross-section of the conductor to the right side is Equal to the no. of Electrons (-ve charges) crossing the cross-section of the cond. to left side. The net flow of -ve charges through the given cross-section of the cond. is zero. There is no electric current in the conductor.

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Let us apply electric field $E = \left(\frac{V}{l}\right)$ across the conductor by connecting its ends to battery. V is the potential difference across the ends of the conductor of length $l$. Each electron experience a force $\vec{F} = -e\vec{E}$ in the direction opposite to the direct^n of applied electric field. The free electrons ~~are~~ in the conductor are accelerated in the direct^n of opposite to the direct^n of applied electric field. The accelerated electron remains in motion until it colloids with an ion in the crystal lattice of the conductor. After collision, the electrons again accelerated under influence of applied electric field and comes to rest after colloiding with ion of the crystal lattice. The electrons in the conductor drift from one end of the conductor to other end of the conductor under the applied electric field with the drift velocity of the order $10^{-4}\ ms^{-1}$.

# CURRENT DENSITY:-

The current density at any point defined as the current flowing per unit area normal to the direct^n of current at that point. This is called

Current Density. The direcⁿ of current density is $\vec{J}$ is the directⁿ of positive charge at that point. An Electron at that point would move in the directⁿ of $\vec{J}$. If



If current I is uniformly distributed across the uniformly cross-sectional area S, then the magnitude of current density for all points on that cross-section is given by

$$J = \frac{I}{S} \quad \text{--(1)}$$

General relationship betⁿ $\vec{J}$ and I for a particular surface in a conductor is given by

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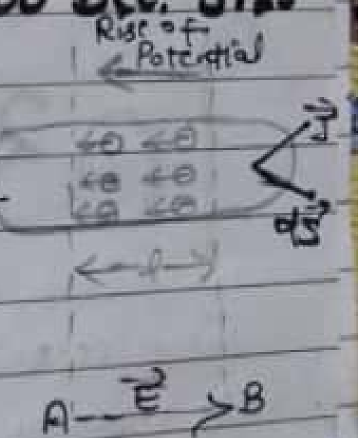
$$I = \iint_S \vec{J} \cdot d\vec{s} \quad \text{--(2)}$$

$d\vec{s}$ is the surface area of the Element of the Conductor and integral is taken over the whole surface.

**Eqn. (2) is show clearly that I is scalar bec. $\vec{J} \cdot d\vec{s}$ is a scalar.**




## Units of current density:-

From Eqn. (i)

$$J = \frac{I}{S}$$

$$\text{Units of current density} = \frac{\text{Units of current}}{\text{units of area}}$$

$$\text{,,} = \frac{A}{m^2} = A m^{-2} \text{ Ans.}$$

**Drift Velocity:-** It is defined as that average velocity with which free Electrons in a Conductor drift in the direct^n of opposite the direct^n of applied Electric field.

Consider a Conductor of length l whose Ends Connected of battery of Voltage V. The Electric field $E = \frac{V}{l}$ is set up across the Ends of the Conductor.

The Force acting on the Electrons in the Conductor is given by, $\vec{F} = -e\vec{E}$. -ve sign shows that the direct^n of Force on Electron is opposite to the direct^n of the Electric field. The acceleration is produced in the Electrons,

$$\vec{a} = \frac{\vec{F}}{m}$$

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$$\vec{a} = \frac{-e\vec{E}}{m} \quad \{\vec{F} = -e\vec{E}\}.$$

The accelerated Electron gain extra Velocity but this velocity is destroyed at Each Collision bet^n Electron and ion in the Conductor. At this stage Electron again accelerated by the electric force from its rest position. The average initial Velocity of Electron is Zero.

Average Velocity of all Electrons is given by:-

$\vec{v_d} = \vec{u} + \vec{a}t$ —— ②, $t$ is the average time betⁿ two successive collisions and is known as relaxation time.



Let $\tau$ the time taken by the electron to achieve velocity $\vec{v_d}$, then $t = \frac{0+\tau}{2} = \frac{\tau}{2}$

But $\vec{u} = 0$ and $\vec{a} = -\frac{e\vec{E}}{m}$



i.e. Eqn. ② becomes

$$\vec{v_d} = 0 - \frac{e\vec{E}\tau}{2m} \Rightarrow \vec{v_d} = -\frac{e\vec{E}\tau}{2m} \quad —③$$

Drift velocity is the velocity with which electrons are drifted through the conductor under influence of an external field.

## * Stationary or Steady Currents — Continuity Eqn.:—

The relation betⁿ current and current density is

$$I = \iint_S \vec{J} \cdot d\vec{s} = \frac{dq}{dt}$$

If the value of current density $\vec{J}$ remains unchanged with time, the current is said to be steady.

Consider a closed surface enclosing volume $V$. If $\rho$ be the charge density of volᵐ $v$, then infinitesimally volᵐ $dv$, $\iiint_V \rho dv$ represents the total charge inside the volᵐ $v$.

According to the law of conservation of charge, the rate of flow of charge through the enclosed surface is equal to the rate of decrease of charge in it.

$$\oiint_S \vec{J}\cdot d\vec{s} = -\frac{d}{dt}\iiint_V \rho\, dv = -\iiint_V \left[\frac{\partial \rho}{\partial t}\right] dv$$

Acc. to Gauss's divergence theorem

$$\oiint_S \vec{J}\cdot d\vec{s} = \iiint_V (\vec{\nabla}\cdot\vec{J})\, dv \quad \text{— (ii)}$$

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From relation (i) and (ii) we get

$$\iiint_V (\vec{\nabla}\cdot\vec{J})\, dv = \iiint_V \left[-\partial \rho/\partial t\right] dv.$$

Above relation holds good for arbitrary volume

So, $\vec{\nabla}\cdot\vec{J} = -\dfrac{\partial \rho}{\partial t} \Rightarrow \vec{\nabla}\cdot\vec{J} + \partial\rho/\partial t = 0$ — (iii)

Eqn (iii) is called the **Eqn. of continuity** and represents the physical fact of conservation of charge.

When steady current flows through the conductor the electric charge do not accumulates at any point on the conductor. This means that the total amount of charge entering volume V through cross-section A is same as the total of charge leaving the volume V through se

There is no change in the charge density in the Volume $v$ or $\frac{\partial \rho}{\partial t} = 0$

$\therefore$ from Eqn. (iii) $\vec{\nabla}\cdot\vec{J} = 0$ for steady currents.

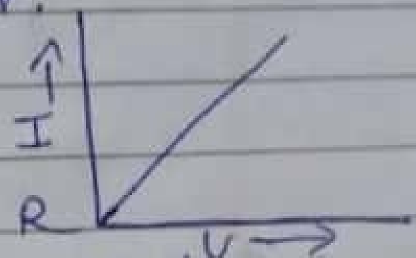
$$\boxed{\vec{\nabla}\cdot\vec{J} = 0} \quad (iv).$$



**OHM'S Law:—** The potential diff. betn. the ends of the conductor is directly proportional to the current flowing through it, is called ohm's law.

Potential diff. $\propto$ Current.

$$\frac{P.D}{\text{current}} = \text{Constt.}$$




This Constt. is called the resistance R. If $V$ be the potential diff. across of the conductor through which current the ends of the conductor I flows.

$$\Rightarrow \boxed{R = \frac{V}{I}}$$

- **Resistance:—** It may be defined as that the ratio of potential diff. ($V$) betn. these points and current $I$, flowing through the conductor.

$$R = \frac{V}{I} \quad \text{General form of ohm's law.}$$

**Unit of Resistance :-** SI unit of resistance is ohm $\Omega$.

- **Resistivity:–** Resistivity is a characteristic of material of conductor. It may be defined as that the ratio of Electric field intensity to current density.

$$\rho = \frac{\text{Electric field intensity}}{\text{Current density}} = \frac{E}{J}$$

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Unit of Resistivity. SI unit $\rho = \frac{\Omega \times m^2}{m}$
$= \Omega m$

Definition of Resistivity $\rho$:

$$\rho = R.\frac{A}{l} \Rightarrow l = 1 \text{ unit}$$
$$A = 1 \text{ unit}$$
$$\Rightarrow \rho = R.$$

- **Conductance and Conductivity:–** The reciprocal of resistance of known as conductance. Conductance is represented by G.

SI unit of Conductance is $\Omega^{-1}$ (mho) is called Simen.

$$G = 1/R = \frac{1}{\rho . l/A} = \frac{1}{\rho}.\frac{A}{l} = \sigma.\frac{A}{l}$$

i.e $\sigma = \frac{1}{\rho}$ is termed as the Conductivity.

**SI unit of Conductivity:–**

$$G = \sigma.\frac{A}{l}$$

$$\sigma = G.l/A = \text{Simen} \times \frac{\text{metre}}{\text{metre}^2}$$

$$= \frac{\text{Simen}}{\text{metre}} \quad Sm^{-1}$$

# CHAPTER-5
## Magnetism:

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**Magnetic field:-** The space around a magnet within which its effect can be experienced. This field is known as magnetic field. It is denoted by ($\vec{M}$).

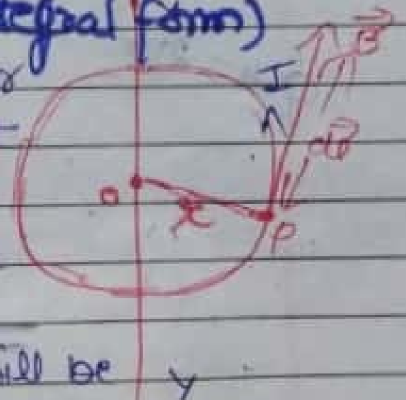
## Ampere's Circuital law:-

**Ampere's Circuital law:-** It states that the line integral of magnetic field ($\vec{B}$) around any closed circular path is equal to $\mu_0$ (times) (permeability of free space). The total current I threading the closed circuit is called Ampere's circuital law.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Integral form:

- To prove the Ampere's circuital law for circular path in (Integral form)




Consider, a long straight conductor XY. Let point P be any point on the conductor. Current flows through the cond. X and Y due to current.

Magnetic field will be produced anticlockwise the directⁿ conductor.

Magnetic field will be produced when the current flows through the wire.

Acc to Right hand rule:-

Magnetic field at point P be tangent to the circular path.

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$$\oint \vec{B}\cdot d\vec{l} = \oint B\,dl\cos 0°$$
$$= B\oint dl$$
$$= \oint B\,dl \quad \text{--- (1)}$$

, $\vec{B}$ and $d\vec{l}$ are same in direcn.

Now, Magnetic field due to current carrying straight conductor.

$$\Rightarrow \boxed{B = \frac{\mu_0}{4\pi}\cdot\frac{2I}{x}}$$

Substituting the value of magnetic field is given by :—

$$\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = \oint \frac{\mu_0}{4\pi}\frac{2I}{x}\cdot dl$$

$\oint dl = 2\pi x$

$$= \frac{\mu_0}{4\pi}\cdot\frac{2I}{x}\oint dl$$
$$= \frac{\mu_0}{4\pi}\cdot\frac{2I}{x}\times 2\pi x$$

$$\Rightarrow \boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I}$$

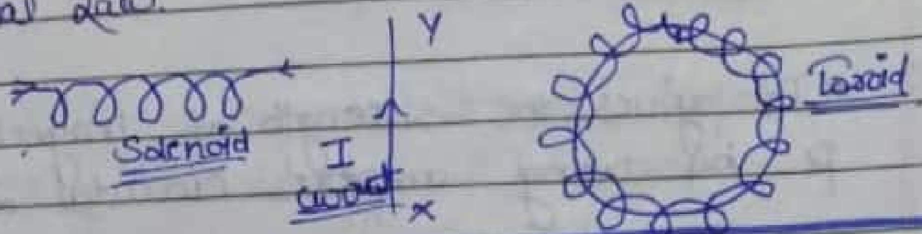
The line integral of magnetic field $\vec{B}$ around closed path bound current is Independent of path.

- If the closed path does not bound any current

$$\Rightarrow \boxed{\oint \vec{B}\cdot d\vec{l} = 0}$$

110 Applications of Ampere's circuital law:-

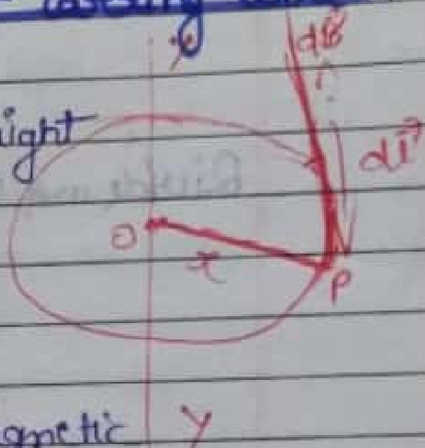
- The ampere's circuital law can be used to derive the expression of magnetic field due to straight line, solenoid and toroid etc. Uses of ampere's circuital law.




## Magnetic field due to current carrying wire

Consider an infinitesimally long straight wire XY. suppose P is at point on the wire at dist$^n$ $x$ from the centre of the wire.

magnetic field




We know that, the magnetic field will be produced, when current flows through the conductor.

Ampere's circuital law.

$\oint \vec{B}\cdot d\vec{l}$ = line integral of magnetic field along circular path.

⇒ Acc. to Ampere's circuital law,

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I} \quad \text{—(1)}$$

$$\oint \vec{B}\cdot d\vec{l} = \oint B\,dl\cos 0^\circ$$

$$= B\oint dl$$

$$= B.\,2\pi x.$$

$$\oint \vec{B}\cdot d\vec{l} = B.\,2\pi x. \quad \text{—(2)}$$

$= B.\ 2\pi x$

$= \frac{\mu_0}{4\pi}\cdot\frac{2I}{x}\cdot 2\pi x = \mu_0 I$

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I}$$

• It gives the strength of magnetic field at P by using ampere's circuital law.

putting eqn. (i) and (ii) we get

$\Rightarrow B.\ 2\pi x = \mu_0 I \Rightarrow B = \frac{\mu_0}{2\pi}\frac{I}{x}$

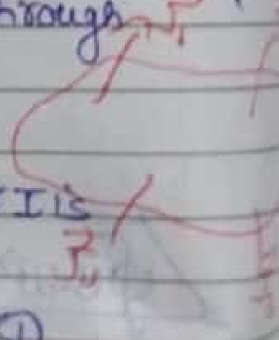
Divide and Multiply by 2 ~~b.s.~~ we get.

$$\Rightarrow B = \frac{\mu_0}{4\pi}\frac{2I}{x}$$

$$\Rightarrow \boxed{B = \frac{\mu_0}{4\pi}\frac{2I}{x}}$$ Ans.

## Curl of a magnetic field (Ampere's circuital law) (Diff. form) ~~Integral form~~:—

Consider a region of space flowing through the current. The current density $\vec{J}$ varies from point to point times.



So, The total steady current I is given by,

$$I = \iint_S \vec{J}\cdot d\vec{s}. \quad \text{——— (1).}$$

Where, $\vec{J}$ is the current density and area of an Element of the surface S,

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$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \quad \text{--- (2)}$$

put I in Eqn. (2).

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \iint_S \vec{J}\cdot d\vec{s} \quad \text{--- (3)}$$

⇒ Acc. to stoke's theorem apply in $\vec{B}$ eqn.

$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \iint_S (\vec{\nabla}\times\vec{B})\, d\vec{s} \quad \text{--- (4)}$$

⇒ Eqn. (4) is put in (3) Eqn.

$$\Rightarrow \iint_S (\vec{\nabla}\times\vec{B})\, d\vec{s} = \mu_0 \iint_S \vec{J}\cdot d\vec{s}$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}}$$

$$\boxed{\text{curl } \vec{B} = \mu_0 \vec{J}}$$

i.e which is the differential form of Ampere's law. Ans.

- Magnetic field due to current carring wire.

(i) Magnetic field outside the wire:—


Let us consider a straight wire AB carring current I. The magnetic field lines are concentric circles centered on the wire in planes ⊥ar to wire.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \Rightarrow \oint B\,dl\cos 0 = \mu_0 I$$

113 $$\Rightarrow B\oint dl = \mu_0 I \Rightarrow B.2\pi x = \mu_0 I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi x}} \quad \text{--- (1)}$$

(ii) Magnetic field at the surface:—

If $x = R$

Now, $B = \dfrac{\mu_0 I}{2\pi R}$ — (2).

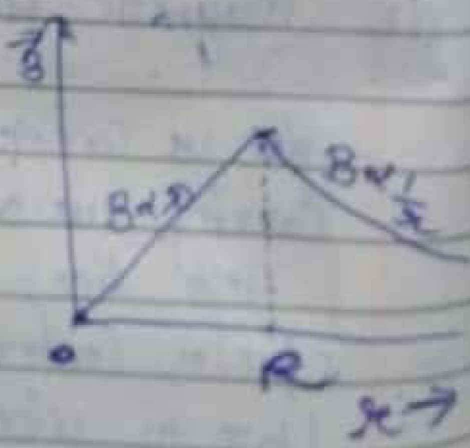
(iii) Magnetic field at a point inside the wire:—


Point p is inside the wire, then symmetry suggests that $\vec{B}$ is tangent to the path of concentric circles. If, the current is steady and uniformed distributed uniformly through the cross-section of the wire.

$$I' = \frac{I}{\pi R^2} \times \pi x^2 = \frac{Ix^2}{R^2}$$

From Ampere's circuital law,

$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 I'$$




$$\Rightarrow B.2\pi x = \mu_0 \frac{Ix^2}{R^2}$$

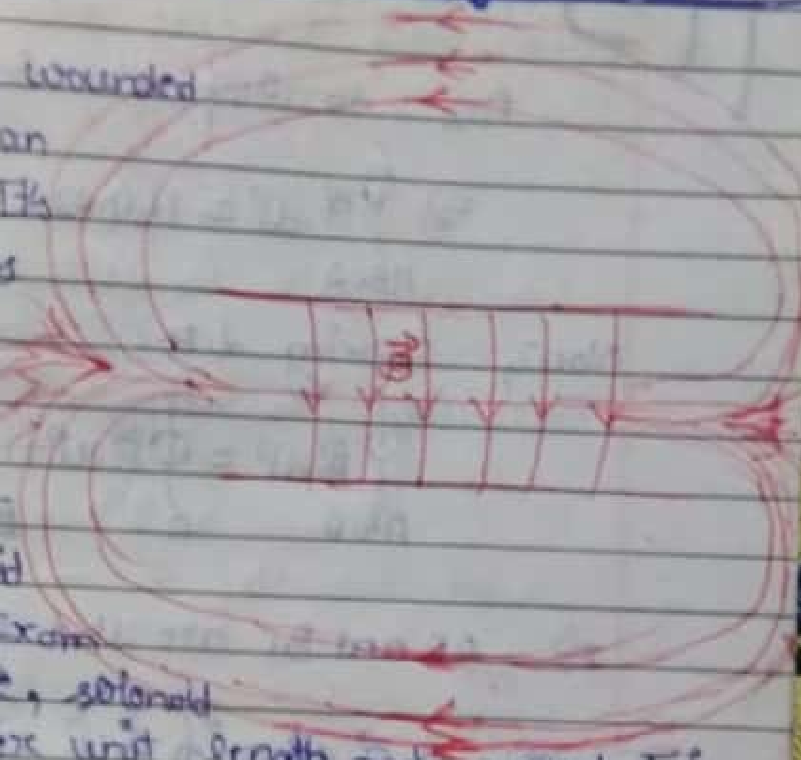
$$\Rightarrow B = \mu_0 \frac{Ix^{\cancel{2}}}{2\pi \cancel{x} R^2}$$

$$\Rightarrow \boxed{\vec{B} = \mu_0 \frac{Ix}{2\pi R^2}}$$

$$B = \frac{\mu_0 I x}{2\pi R^2} \quad \text{—(3)}$$

# 114 Magnetic field due to a Long Solenoid

A Solenoid is tightly wounded helical loop from an ~~fed~~ insulated wire. It's length is larger as compared to its diameter are called Solenoid.

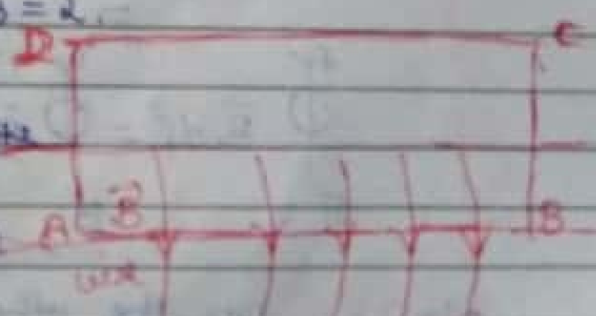



Consider a Solenoid having $n$ turns from an insulated wire, solenoid having $n$ turns per unit length and current I is passed through the magnetic field.

It may be also noted that, the magnetic field is produced by solenoid inside it is parallel to the (solenoid) length of the Solenoid. To find, the magnetic field inside the solenoid to take rectangular path from ABCD.

Such that, $AB = l$.

The no. of turns of the Solenoid is Enclosed by the rectangle is equal to $n l$.




$\Rightarrow$ The current flows through it. Total current

Threading the solenoid.

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$AB = l$.

Total current $= (nlI)$. —— (1).

Acc. to Ampere's circuital law,

$$\Rightarrow \oint_{ABCD} \vec{B}\cdot\vec{dl} = \mu_0 (nlI) \text{ —— (1).}$$

Now, Taking L.H.S.

$$\oint_{ABCD} \vec{B}\cdot\vec{dl} = \oint_A^B \vec{B}\cdot\vec{dl} + \oint_B^C \vec{B}\cdot\vec{dl} + \oint_C^D \vec{B}\cdot\vec{dl} + \oint_D^A \vec{B}\cdot\vec{dl}$$

$\Rightarrow$ AD and BC are ⊥ar to each other, then

$$\Rightarrow \oint_{ABCD} \vec{B}\cdot\vec{dl} = \oint_A^D \vec{B}\,\vec{dl} + \oint_B^C \vec{B}\cdot\vec{dl}$$

$$= \oint_A^D B\,dl\cos 90^\circ + \oint_B^C B\,dl\cos 90^\circ$$

$$= \cancel{B}\oint_A^D dl + \cancel{B\oint_B^C dl}\; 0 + 0$$

$$= 0.$$

| | 0 | 30° | 45 | 60° | 90° |
|---|---|---|---|---|---|
| sin | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| cos | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |

$$\oint_D^A \vec{B}\cdot\vec{dl} = \oint_B^C \vec{B}\cdot\vec{dl} = 0.$$

Now, on the other hand.

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$$\oint_{ABCD} \vec{B}\cdot\vec{dl} = \oint_{A}^{B} \vec{B}\cdot\vec{dl} + \oint_{C}^{D} \vec{B}\cdot\vec{dl}$$

⇒ Solenoid whose length is Equal to its diameter. But, the magnetic field outside the solenoid is practically zero.

Now,
$$\oint_{ABCD} \vec{B}\cdot\vec{dl} = \oint_{C}^{D} \vec{B}\cdot\vec{dl} = 0$$

Now,
$$\oint_{ABCD} \vec{B}\cdot\vec{dl} = \oint_{A}^{B} \vec{B}\cdot\vec{dl} = \oint_{A}^{B} B\,dl\cos 0^\circ$$

$\left(dl = L,\ \oint dl = 2\pi x\right)$

$$= B\int_{A}^{B} dl = BL$$

$$\oint_{ABCD} \vec{B}\cdot\vec{dl} = BL \quad \text{—(5)}$$

Put (5) in Eqn. (1) we get

$$B\cancel{L} = \mu_0 (n\cancel{L} I)$$

$$\Rightarrow \boxed{B = n\mu_0 I} \text{ Ans}$$

## Magnetic Field due to a toroidal solenoid :-


Consider a toroidal solenoid Consis-ting an anchor ring at point O.

Let n be no. of turns per unit length out a solenoid wound of the ring.

Suppose that, Current I is flowing through the to

Magnetic field is ~~same~~ same at all the points on the circumference on the ring and it's direction is tangent to the circular ring.

117 Acc. to Ampere's Circuital law:-

$\oint \vec{B}\cdot d\vec{l} = \mu_0 \times q I$ current flowing through the toroid / the solenoid.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 (2\pi r n I) \quad \text{--- (1)}$$

Taking L.H.S.

$$\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = B\oint dl$$

$$\oint \vec{B}\cdot d\vec{l} = B\,2\pi r \quad \text{--- (2)}$$

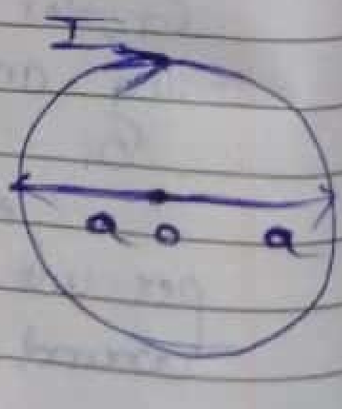
$\Rightarrow$ put (2) eqn. in (1) eqn.

$$B\cdot \cancel{2\pi r} = \mu_0 (\cancel{2\pi r} n I)$$

$$\Rightarrow \boxed{B = \mu_0 n I} \text{ Ans.}$$

## Magnetic field due to portion of circular coil, Carring Current

Magnetic field due to ½ portion of the Coil.




$$B = \frac{\mu_0}{4\pi} \times \frac{2\pi I}{a}$$

$$\Rightarrow \boxed{B = \frac{1}{2}\frac{\mu_0}{4\pi} \times \frac{\pi I}{a}}$$

which gives the acceleration in rotating frame → multiplying (10) by m from

$m\vec{a}_R = m\vec{a}_S - 2m(\vec{\omega}\times\vec{v}_R) - m\vec{\omega}\times(\vec{\omega}\times\vec{r})$

mass × acc = F $\Rightarrow \boxed{\vec{F}_R = \vec{F}_S + \vec{F}_{cor} + \vec{F}_{centri}}$



Here, * $\vec{F}_{cor} = -2m(\vec{\omega}\times\vec{v}_R)$ is the Coriolis force
Coriolis force and

$\Rightarrow m\vec{a}_S$ is the force

$\vec{F}_{centri} = -m\vec{\omega}\times\vec{\omega}\times\vec{r}$ is the Centrifugal force. Ans

Physics:-

## Hall EFFECT :-

When a conductor (metal or semiconductor) carrying current is placed in transverse magnetic field, an electric field is produced inside the conductor in direction normal to both current and magnetic field. This is known as Hall effect.

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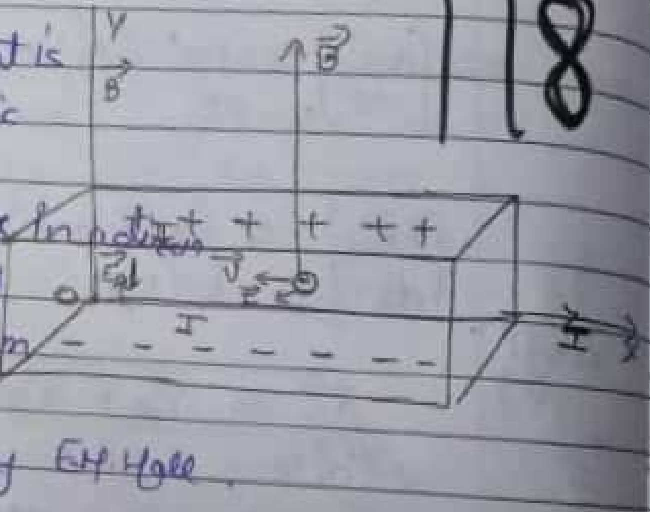



It was dis. by EH Hall.

Hall Effect helps us to know about the nature of charge carriers.

Case I:-

Negatively charged particles:-

Consider a conductor through which the ~~conduct~~ current I flows along with positive x-axis. The negatively charged particles are electrons (-e). Let a uniform magnetic field $\vec{B}$ acts along in direct^n of Y-axis.

Then each electron experiences a magnetic force is given by:-

$\vec{F} = -e(\vec{v}\times\vec{B})$,

$\vec{v}$ = Drift velocity of electrons

Acc to fleming's left hand rule, the direct of force $\vec{F}$ acts along z-axis. The current carriers Electrons are deflected towards z-axis.

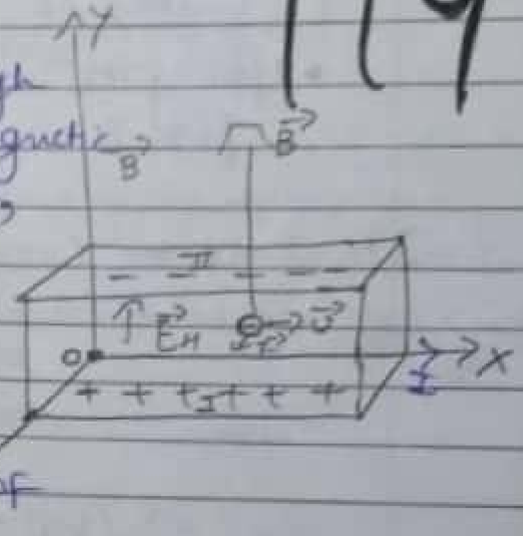
Transverse p.d are called Hall voltage

⇒ Transverse potential diff on electric field is called Hall field.

Case II :—

Positively charged particles:—

Let I current is flowing through the cond. along x-axis, and magnetic field $\vec{B}$ acts along y-axis. If, the current carriers are holes.




Each hole experience a force i.e

$$\vec{F} = +e(\vec{v} \times \vec{B})$$

$\vec{v}$ = drift velocity of the hole.

⇒ Acc. to fleming's left hand rule, the direct^n^ of the force $\vec{F}$ is along z-axis. They will move towards face I. face I becomes positively charged and face II becomes -vely charged.

Conclusion:— (i) If the face I is -vely charged due to flow of current through the specimen along x-axis, current carriers are Electrons, given specimen is conductor.

(ii) If face I is +vely charged due to flow of current through the specimen along x-axis, current carriers are holes, given specimen is semiconductor.

*Hall Effect [Expression for Hall Constant:]

$\vec{E_H}$ is the Hall field along z-direct^n^. Current carriers are 'e' is $eE_H$.

Magnetic force on the current carriers $e(\vec{v_d} \times \vec{B})$.

Net force on the current carrier is zero:-

$$eE_H + e(\vec{v_d} \times \vec{B}) = 0$$

$$\vec{E_H} = -\vec{V_d} \times \vec{B}$$

i.e. $\vec{B}$ is $\perp$ to $\vec{V_d}$

$$E_H = -v_d B \sin\theta$$

$$E_H = -v_d B \sin 90^\circ$$

$$\Rightarrow \boxed{E_H = -v_d B} \quad \text{--- (I)}$$

- Hall field is directly proportional to magnetic field B.

If J be the current density in the x-directⁿ.

$$J = nev_d$$

$\Rightarrow n =$ conc. of current carriers.

$\Rightarrow V_d =$ Drift velocity of charge carriers.

Hall voltage is given by

$$\Rightarrow \boxed{V_d = \frac{J}{ne}} \quad \text{--- (ii)}$$

- d is the distⁿ betⁿ face I and II.

$$\Rightarrow V_H = E_H d \Rightarrow E_H = \frac{V_H}{d}$$

Using eqn (I) $E_H = E_H d = -v_d B d \Rightarrow v_d = E_H/B$

Substituting this value in eqn. (ii).

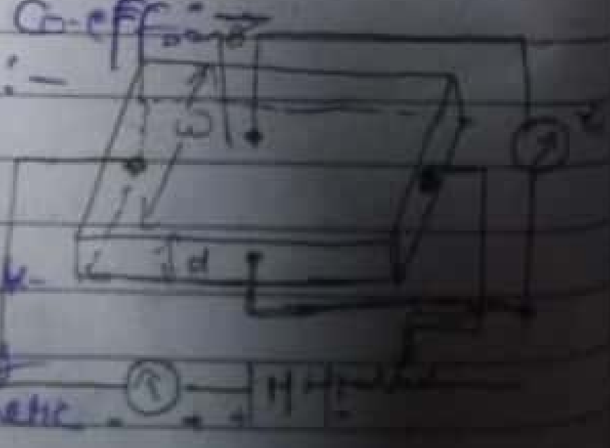
$$\frac{E_H}{B} = \frac{J}{ne} \quad \text{or} \quad \frac{E_H}{JB} = \frac{1}{ne} = R_H$$

i.e. $R_H$ is called Hall constant or Hall coefficient.

3.* Determination of Hall Co-eff:-

Hall co-eff. is given by:-

$$R_H = \frac{E_H}{JB} \quad \text{--- (I)}$$



Cond. of width ω and thickness d connected with battery is placed in uniform magnetic field $\vec{B}$.

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Area of cross-section of cond. $A = d \times \omega$.

$\Rightarrow$ current density $J = \frac{I}{A} = \frac{I}{d \times \omega}$

$$\Rightarrow \boxed{E_H = \frac{V_H}{d}}$$

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(1) Eqn. becomes :—

$$R_H = \frac{\frac{V_H}{d}}{\frac{I}{d \times \omega} \times B} \Rightarrow R_H = \frac{V_H}{\cancel{d}} \times \frac{\cancel{d} \times \omega}{I B}$$

$$\Rightarrow \boxed{R_H = \frac{V_H \omega}{I B}}$$

*knowing the values of $V_H$, $\omega$, I and B, we can determine the Hall co-eff ($R_H$) Ans

*Applications of Hall Effect :—

i) Determination of carrier concentration :—

$$\underset{\text{Hall Coeff.}}{R_H} = \frac{1}{ne}$$

$$\boxed{n = \frac{1}{R_H \cdot e}}$$

(ii) Determination of type of semiconductor :—

$\boxed{R_H = \frac{1}{ne}}$, For n-type semicond. ~~carrier con~~ current carriers are electrons, $R_H$ will be -ve and p-type semi-cond. current carriers are holes, $R_H$ will be +ve.

Uses of Hall Effect :—

(i) It is used to find the nature of charge carriers.

(ii) It is used to find the no. of charge carriers. Ans

# Divergence of Magnetic field Vector $\vec{B}$.

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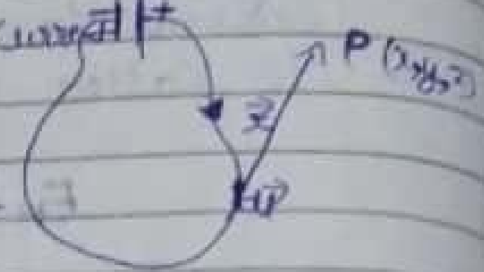
• Diff. Form of Gauss's law in magneto-statics

Magnetic field produced by a current Element I at a point $p(x, y, z)$.




Acc. to Biot'z Swatz' law:-

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \vec{x}}{x^3}.$$

Magnetic field at point p due to whole Current loop:-

$$\oint d\vec{B} = \oint \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{x}}{x^3}.$$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} I \oint \frac{d\vec{l} \times \vec{x}}{x^3}} \quad \text{--- (1)}$$

Taking divergence on both sides, we get

$$\vec{\nabla}.\vec{B} = \frac{\mu_0}{4\pi} I \oint \vec{\nabla}. \left[\frac{d\vec{l} \times \vec{x}}{x^3}\right].$$

$$\Rightarrow \boxed{\vec{\nabla}.\vec{B} = \frac{\mu_0}{4\pi} I \oint \vec{\nabla}. \left[\frac{d\vec{l} \times \vec{x}}{x^3}\right]} \quad \text{--- (a)}$$

$$\vec{\nabla}.(\vec{A} \times \vec{B}) = \vec{B}.(\vec{\nabla} \times \vec{A}) - \vec{A}.(\vec{\nabla} \times \vec{B})$$

Now, $\vec{\nabla}.\vec{B} = \frac{\mu_0}{4\pi} I \left[\oint \left(\frac{\vec{x}}{x^3}\right).\left[\vec{\nabla} \times d\vec{l}\right) - d\vec{l}.\left(\vec{\nabla} \times \frac{\vec{x}}{x^3}\right)\right]$

$dl'$ is not function of $x, y$ and $z$ so $\vec{\nabla} \times d\vec{l'} = 0$.

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i.e. $\vec{\nabla} \times \frac{\vec{r}}{r^3} = -\vec{\nabla} \times \vec{\nabla}\left(\frac{1}{r}\right)$ ——— (4)

$\left\{ \frac{\vec{r}}{r^3} = -\nabla\left(\frac{1}{r}\right) \right.$

Curl of gradient is zero.

$$\vec{\nabla} \times \frac{\vec{r}}{r^3} = -\vec{\nabla} \times \vec{\nabla}\left(\frac{1}{r}\right) = 0 \quad \text{——— (5)}$$

Using eqn. (4) and (5) eqn. (3) becomes

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0} \quad \text{——— (6)}$$

Divergence of $\vec{B}$ is zero.

A vector whose divergence is zero is called Sole-noid. Eqn. (6) is called differential form of Gauss's law in magnetostatics.

## Diff. betn. Electrostatic field and Magnetostatic field.

| Electrostatic field | Magnetostatics field |
| --- | --- |
| (i) Electrostatic field is due to charges ~~of~~ at in rest or in motion. | Magnetostatics / Magnetic field is due to only in motion |
| (ii) Line integral of ~~magn~~ Electric field along a closed path is zero. $\oint \vec{E} \cdot d\vec{l} = 0$ | Line integral of magnetic field is equal to $\mu_0$ times the total current threading the circuit. $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ |

(iii) Curl of $\vec{E} = \vec{\nabla} \times \vec{E} = 0$ Curl $\vec{B} = \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

(iv) div $\vec{E} = \vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$ $\boxed{\text{div } \vec{B} = \vec{\nabla} \cdot \vec{B} = 0}$

(v) It is conservative in nature. It is non conservative in nature

Q1:- What do you understand by toroid?

Ans:- Toroid:— A toroid is just a solenoid of finite length bent into a circle so that it has no ends. It is also called anchor ring.

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## Magnetic Vector Potential:—

Acc to magnetostatics ie,

$\vec{\nabla} \cdot \vec{B} = 0$. — (1)

If $\vec{A}$ is another vector such that

$\boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$

$\Rightarrow \vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot \vec{\nabla} \times \vec{A}$. — (2)

divergence of curl is always zero

$\vec{\nabla} \cdot \vec{\nabla} \times \vec{A} = 0$. — (3)

Comparing eqn (1) and (3),

$\boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$

Vector $\vec{A}$ is known as magnetic vector potential

# Expression ~~of~~ for Magnetic Vector Potential ($\vec{A}$).

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Acc. to Biot savart's law,

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2} \quad \text{--- (1).}$$

Instead of current element, consider a vol^m distribution of current. we can write $dv = s\,dl$ / for a vol^m element

$$\left(\frac{I}{s} = \vec{J}\right)$$

$$I\,d\vec{l} = \frac{I}{s} s\,d\vec{l} = \vec{J}.s\,dl = \vec{J}\,dv.$$

Eqn (1),

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{J}\,dv \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\left(\frac{\vec{J} \times \hat{r}}{r^2}\right) dv$$

Magnetic field $\vec{B}$ at any point at a dist^n $r$ from the vol^m distribution of current of vol^m $dv$

$$\vec{B} = \frac{\mu_0}{4\pi} \iiint_V \left(\frac{\vec{J} \times \hat{r}}{r^2}\right) dv$$

$$\vec{B} = -\frac{\mu_0}{4\pi} \iiint_V \vec{J} \times \vec{\nabla}\left(\frac{1}{r}\right) dv$$

$\Rightarrow$ Vector identity :-

$$-\vec{J} \times \vec{\nabla}\left(\frac{1}{r}\right) = \vec{\nabla}\left(\frac{1}{r}\right) \times \vec{J} = \vec{\nabla} \times \left(\frac{\vec{J}}{r}\right) + \left(\frac{1}{r}\right)\vec{\nabla}$$

for steady current :- $\boxed{\vec{\nabla} \times \vec{J} = 0}$

$$-\vec{J}\times\vec{\nabla}\left(\frac{1}{r}\right) = \vec{\nabla}\times\left(\frac{\vec{J}}{r}\right)$$

Eqn. ② becomes

$$\vec{B} = \frac{\mu_0}{4\pi}\iiint_V \vec{\nabla}\times\left(\frac{\vec{J}}{r}\right) dv$$

$$\vec{B} = \vec{\nabla}\times\frac{\mu_0}{4\pi}\iiint_V\left(\frac{\vec{J}}{r}\right) dv.$$

$$\Rightarrow \vec{B} = \text{Curl}\,\vec{A} = \vec{\nabla}\times\vec{A}.$$

$$\vec{\nabla}\times\vec{A} = \vec{\nabla}\times\frac{\mu_0}{4\pi}\iiint_V\left(\frac{\vec{J}}{r}\right) dv.$$

$$\Rightarrow \boxed{\vec{A} = \vec{\nabla}\times\frac{\mu_0}{4\pi}\iiint_V\left(\frac{\vec{J}}{r}\right) dv}$$

i.e where $\vec{A}$ is called **vector potential.**

## [Divergence of $\vec{A}$.]

$\vec{A} = \frac{\mu_0}{4\pi}\iiint_V\left(\frac{\vec{J}}{r}\right) dv$

Acc. to the definition of vector potential:-

$$\vec{A} = \frac{\mu_0}{4\pi}\iiint_V\left(\frac{\vec{J}}{r}\right) dv$$

Taking the divergence of b.s,

$$\vec{\nabla}\cdot\vec{A} = \vec{\nabla}\cdot\frac{\mu_0}{4\pi}\iiint_V\left(\frac{\vec{J}}{r}\right) dv$$

$$= \frac{\mu_0}{4\pi}\iiint_V \vec{\nabla}\cdot\left(\frac{\vec{J}}{r}\right) dv \;\text{----}\; (4)$$

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Acc. to Gauss's divergence theorem:-

$$\iiint_V \vec{\nabla}\cdot\left(\frac{\vec{J}}{r}\right) dv = \iint_S \left(\frac{\vec{J}}{r}\right) d\vec{s}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\mu_0}{4\pi} \iint_S \left(\frac{\vec{J}}{r}\right) d\vec{s} \quad \text{--- (2)}$$

For finite current distribution, $\vec{J} = 0$ at infinity. Since the integral of Eqn (2) is on the bounding surface at infinity,

$$\iint_S \left(\frac{\vec{J}}{r}\right)\cdot d\vec{s} = 0$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{A} = 0} \text{ Ans.}$$

# CHAPTER-6.

## Field of Moving charges

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i.e charge is invariant quantity i.e

$$\oint\!\!\oint_{S(t)} \vec{E}\cdot d\vec{s} = \oint\!\!\oint_{S'(t)} \vec{E'}\cdot d\vec{s'}.$$

The transformation Equations for the Components of electric field is:—

$$E'_x = E_x$$
$$E'_y = E_y$$
$$E'_z = E_z.$$

$$\boxed{\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

**Q1: What is the diff. betn charge invariance and charge Conservation?**

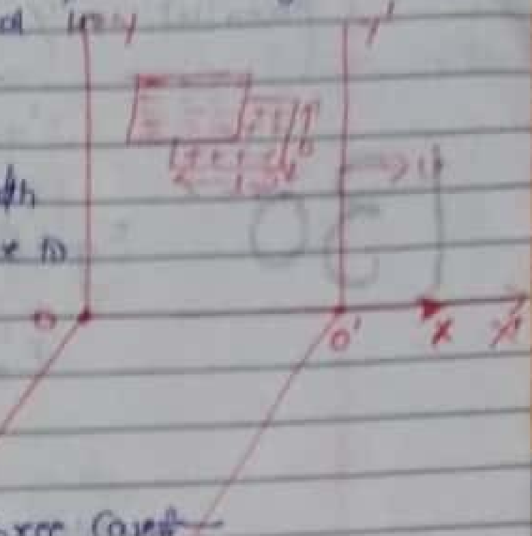
Ans:— Charge invariance means that charge does not change with the motion of charged body called charge invariance and charge conservation means charge of an isolated system remains always Constt.

**Q1:— Derive the relation betn the Components of electric field in laboratory frame and moving frame of reference and show:—**

$$E'_{\parallel} = E_{\parallel} \quad \text{and} \quad E'_{\perp} = \gamma E_{\perp}.$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

**Ans:—** Consider two parallel thin sheet having charge density $\sigma$ and $-\sigma$ is placed in Stationary in a frame of reference S. Let $l$ be the length and $b$ be the breadth
129 of the each plate. We have to take another frame of reference S' moving along positive direction of x-axis with velocity $v$.



We have to here three cases.

**Case I :—** When the sheets are parallel to YZ plane such that the electric field is along x-axis.

Electric field along direct$^n$ of x-axis is,

Since, frame S, the electric field is,

$$E_x = \frac{\sigma}{\epsilon_0}$$

$\left(\sigma = \frac{q}{lb}\right)$

$$\Rightarrow E_x = \frac{q}{lb \times \epsilon_0} \quad \text{——} (1)$$

$\Rightarrow$ Also, in the frame S', if $E'_x$ is

$$\Rightarrow E'_x = \frac{\sigma'}{\epsilon_0} \text{ — } (2),$$ $\sigma$ the charge density on each plate.

$$\sigma' = \frac{q'}{l'b'} = \frac{q}{l'b'}$$ { $q' = q$ due to invariance of charge).

the motion is along x-axis, and $l$ and $b$ are parallel to y and z-axis.

$l' = l$ and $b' = b$.

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Eqn. (2) becomes,

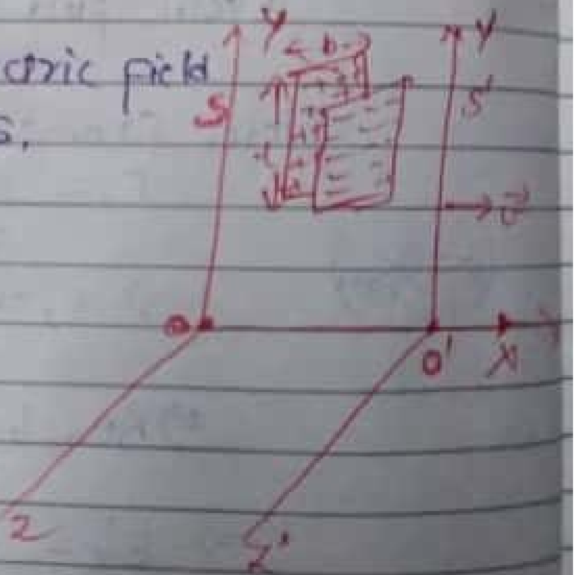
$$E'_x = \frac{\sigma'}{\epsilon_0} = \frac{q}{l'b'\epsilon_0} \Rightarrow E'_x = \frac{q}{lb\epsilon_0}.$$

Now, $\boxed{E'_x = E_x}$ — (3)

**Case II:—** When the sheets are parallel to XZ plane i.e the electric field is along Y-axis.

In frame S:—

det $E_y$ be the electric field as observed in frame S.




$$E_y = \frac{\sigma}{\epsilon_0}.$$

$$E_y = \frac{q}{lb\epsilon_0} \quad \text{— (4)}.$$

In frame S'

det $E'_y$ be the electric field as observed in frame S'.

$$E'_y = \frac{\sigma'}{\epsilon_0}$$

$$\Rightarrow E'_y = \frac{\sigma'}{l'b'\epsilon_0} \quad \text{(5)}$$ since, $b'$ is ~~II~~ to motion of S.

$\boxed{b' = b}$

131 Since $l'$ is lying parallel to x-axis, length contraction occurs,

$$l' = l\sqrt{1 - \frac{v^2}{c^2}} = \frac{l}{\gamma}$$

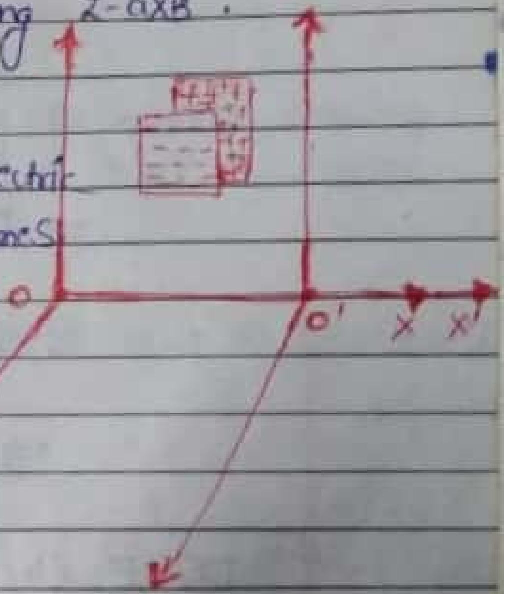
where, $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

Eqn. ⑤ can be written as,

$$E'_y = \frac{q}{\frac{l}{\gamma} b \varepsilon_0} = \gamma \frac{q}{l b \varepsilon_0}$$

$$\Rightarrow \boxed{E'_y = \gamma E_y}$$

Case III :— When the sheets are parallel to XY plane. i.e Electric field is along z-axis.




In frame S :—

Let $E_z$ be the Electric field as observed in frame S

$$E_z = \frac{\sigma}{\varepsilon_0} \Rightarrow E_z = \frac{q}{l b \varepsilon_0}.$$

$$\boxed{E_z = \frac{q}{l b \varepsilon_0}} \quad \text{— ⑥}.$$

In frame S' :—

Let $E'_z$ be the Electric field as observed in frame S'.

$$E'_z = \frac{\sigma'}{\varepsilon_0} = \frac{q}{l' b' \varepsilon_0} \quad \text{— ⑦}$$

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$l$ is $\parallel$ar to $x$-axis, it does not get contracted.

$$l' = l$$

But, since $b$ is $\parallel$ to $y$-axis, length contraction takes place i.e.,

$$b' = b\sqrt{1 - v^2/c^2}$$

$$= b/\gamma \quad \text{where } \gamma = \sqrt{1 - \frac{v^2}{c^2}}$$

Eqn. (1) becomes:-

$$E_z' = \frac{q}{l'b'\epsilon_0} = \frac{q}{\frac{lb}{\gamma}\epsilon_0}$$

$$= \gamma \cdot \frac{q}{lb\epsilon_0} = \gamma E_z$$

$$\Rightarrow \boxed{E_z' = \gamma E_z}$$

$$\Rightarrow E_x' = E_x$$

$$\Rightarrow E_y' = \gamma E_y$$

$$\Rightarrow E_z' = \gamma E_z.$$

If $\vec{E}$ electric field measured in frame S due to charges stationary in it and $\vec{E'}$ is electric field measured in system S' moving with velocity v.

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Electric field along ||lar to v

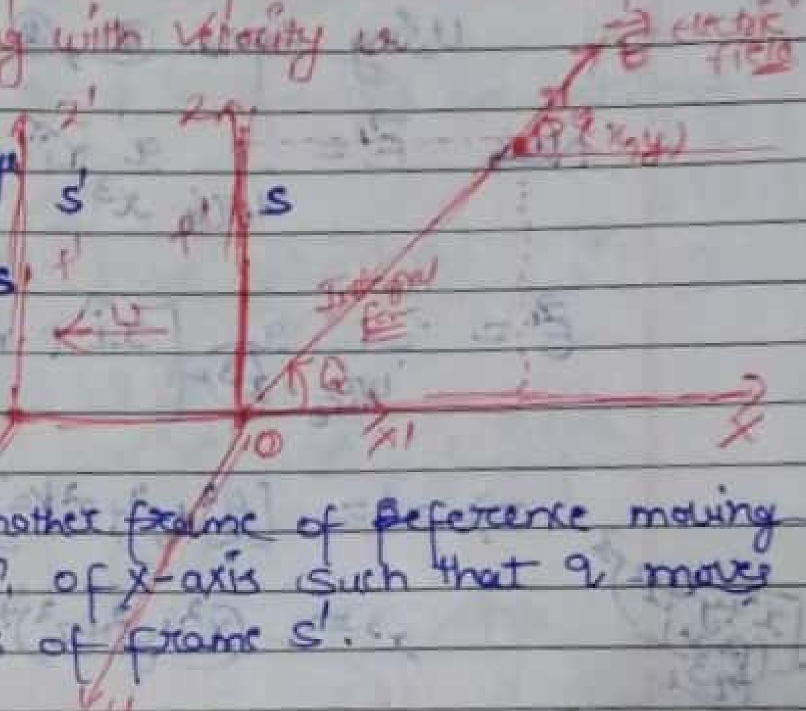
$$E'_{\parallel} = E_{\parallel}$$

$$E'_{\perp} = \gamma E_{\perp}$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

Q23- Derive an Expression for an electric field of a charge moving with velocity v.

Ans:- Consider a charge q is placed in laboratory frames where where the charge is at rest.




Let s' be the another frame of reference moving along +ve direct^n of x-axis such that q moves along -ve x axis of frame s'.

Let $\vec{E}$ be the Electric field due to observed as frame s is,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{x^3}\vec{x} \Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{x^3}(x\hat{i} + y\hat{j} + z\hat{k}).$$

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{q}{x^3}x \quad \text{---} \ (1)$$

$$E_y = \frac{1}{4\pi\epsilon_0}\frac{q}{x^3}y \quad \text{---} \ (2)$$

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{q}{x^3}z. \quad \text{---} \ (3)$$

If $\vec{E}'$ be the Electric field due to $q$ as observed in frame $S'$.

Now, $E'_x = E_x$, $E'_y = \gamma E_y$, $E'_z = \gamma E_z$.

Now, $\vec{E}' = E'_x\hat{i} + E'_y\hat{j} + E'_z\hat{k}$.

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$$\boxed{\vec{E}' = E_x\hat{i} + \gamma E_y\hat{j} + \gamma E_z\hat{k}}.$$

Using Eqns. ①, ② and ③ we get.

Now $\vec{E}' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}x\hat{i} + \gamma\frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}y\hat{j} + \gamma\frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}z\hat{k}$

$$\vec{E}' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\left[x\hat{i} + \gamma y\hat{j} + \gamma z\hat{k}\right] \quad \text{— ④}.$$

$\Rightarrow r^3 = \left[(x^2+y^2+z^2)^{1/2}\right]^3$.

$\left(\sqrt{x^2+y^2+z^2}\right)^3$ $(r^2)^{1/2}$ $(r^3)$

$r^3 = (x^2+y^2+z^2)^{3/2}$.

$x = \gamma x'$, $y = y'$ and $z = z'$.

$r^3 = \left[(\gamma x')^2 + y'^2 + z'^2\right]^{3/2}$ $= \gamma^3\left[x'^2 + \frac{y'^2+z'^2}{\gamma^2}\right]^{3/2}$.

$= \left[(\gamma^2 x'^2)^2 + y'^2 + z'^2\right]^{3/2}$.

$= \left[(\gamma x')^2 + y'^2 + z'^2\right]^{3/2}$.

$r^3 = (\gamma^2 x'^2 + y'^2 + z'^2)^{3/2}$

$\gamma^3\left(x'^2 + \frac{y'^2+z'^2}{\gamma^2}\right)^{3/2}$

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

$\gamma^2 = \frac{1}{1-\frac{v^2}{c^2}} \Rightarrow \gamma^2 = \ldots$

$\boxed{\frac{v}{c} = \beta}$

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$$x^3 = \gamma^3 [x'^2 + (1-\beta^2)(y'^2 + z'^2)]^{3/2}$$

$$= \gamma^3 [x'^2 + y'^2 + z'^2 - \beta^2 (y'^2 + z'^2)]^{3/2}$$

$$= \gamma^3 [r'^2 - \beta^2 (y'^2 + z'^2)]^{3/2}$$

$$x^3 = \gamma^3 x'^3 \left[1 - \beta^2 \frac{(y'^2 + z'^2)}{\gamma'^2}\right]^{3/2} \quad \text{—— (5)}$$

$$\Rightarrow x\hat{i} + \gamma y\hat{j} + \gamma z\hat{k} = \gamma x'\hat{i} + \gamma y'\hat{j} + \gamma z'\hat{k} = \gamma \vec{x}' \quad \text{—— (6)}$$

Using (5) and (6) in Eqn. (4), we get:-

$$\vec{E}' = \frac{1}{4\pi\epsilon_0} \cdot \frac{q\gamma\vec{x}'}{\gamma^3 x'^3 \left[1 - \beta^2 \frac{(y'^2 + z'^2)}{r'^2}\right]^{3/2}}$$

$$\vec{E}' = \frac{1}{4\pi\epsilon_0} \frac{q\vec{x}'}{x'^3 \gamma^2 \left[1 - \frac{\beta^2 (y'^2 + z'^2)}{r'^2}\right]^{3/2}}$$

$$\vec{E}' = \frac{1}{4\pi\epsilon_0} \frac{q(1-\beta^2)\vec{x}'}{x'^3 \left[1 - \frac{\beta^2 (y'^2 + z'^2)}{r'^2}\right]^{3/2}} \qquad \left\{\gamma^2 = \frac{1}{1-\beta^2}\right\}$$

Which gives the Electric field due to moving charges.

Condition:-

(i) $v << c$, $\beta = \frac{v}{c}$.

$\Rightarrow \frac{v}{c} << 1$, $\beta << 1$. Now, we can neglect $(1-\beta^2)$.

$$E' = \frac{Q}{4\pi\epsilon_0 x'^2}$$

(ii) $E' = \frac{Q(1-\beta^2)}{4\pi\epsilon_0 x'^2} \left[\frac{1}{1-\beta^2 \sin^2\theta'}\right]^{3/2}$ Q is dependent.

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If $Q = 0$ charge velocity move (a) $E'_{\parallel} = \dfrac{Q(1-\beta^2)}{4\pi\varepsilon_0 x'^2}$

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(b) $E'_{\perp} = \dfrac{Q(1-\beta^2)}{4\pi\varepsilon_0 x'^2 (1-\beta^2)^{3/2}}$

$\Rightarrow E'_{\perp} = \dfrac{Q}{4\pi\varepsilon_0 x'^2 (1-\beta^2)^{1/2}}$

CHAPTER

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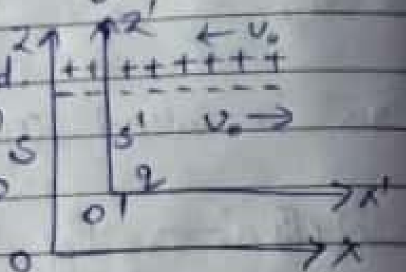
From ① Eqn:—

$$\Rightarrow \sin x = 0 + x(1) + \frac{x^2}{\lfloor 2}(0) + \frac{x^3}{\lfloor 3}(-1) + \cdots \frac{x^{(n-1)}}{\lfloor n-1} \sin\frac{(n-1)\pi}{2} + \frac{x^n}{\lfloor n} \sin\left(\theta x + \frac{n\pi}{2}\right).$$

$$\sin x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \cdots + \frac{x^{n-1}}{\lfloor n-1} \sin\frac{(n-1)\pi}{2} + \frac{x^n}{\lfloor n} \sin\left(\theta x + \frac{n\pi}{2}\right), \quad 0<\theta<1, \; \theta = 1$$

Physics:—

Interaction betn. moving charge and other moving charges:—




Consider two parallel cond. carrying current in the same dirctn. attract each other. Force comes into the play when the charges are in motion. Such forces are called magnetic force.

Ampere put forward hypothesis that a mag. subs contain permanent circulating electric currents. prove that mag. interaction of electric current is corollary of Coulomb's law, If the postulates of relativity are valid.

Interaction betn moving charges, Consider a systm. consisting a long array of positive charge moving towards left with a speed $V_0$ and superimposed on it and array of -ve charge moving towards right with the same speed, $V_0$.

Let the linear charge density of +ve and -ve charges are measured in lab. from be $\lambda_0$. Net charge density of the line in the lab. frame is zero. It means that, Electric field $\vec{E}$ in lab. fram is zero a neutral wire carrying steady current.

In lab frame, the +ve and -ve charges distribution would be contracted (length contraction) x-direct.$^n$ by factor $\left[1-\frac{v_0^2}{c^2}\right]^{1/2}$ which would be very dense in frame in which +ve and -ve charges at rest.

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The l.c.d $\lambda$ in the moving frame of reference is related to $\lambda_0$ by the:—

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$$\lambda=\lambda_0\left(1-\frac{v_0^2}{c^2}\right)^{1/2} \quad \text{—— ①.}$$

If test charge q is at rest, then ~~they~~ it will not experience any force bec$^3$ the $\vec{E}$ is zero.

Consider a frame s' moving with test charge. The mag. of velocity of +ve charges in frame s' is

$$v_+' = \frac{v_0-v}{1-\frac{v_0 v}{c^2}} = \frac{c^2(v_0-v)}{c^2-v_0 v} \quad \text{—— ②.}$$

-ve charges is:—

$$v_-' = \frac{v_0+v}{1+\frac{v_0 v}{c^2}} = \frac{c^2(v_0+v)}{c^2+v_0 v} \quad \text{—— ③.}$$

⇒ linear charge density of +ve charges in s'.

$$\lambda_+' = \frac{\lambda}{\left(1+\frac{v_+^2}{c^2}\right)^{1/2}} = \frac{\lambda_0\left(1-\frac{v_0^2}{c^2}\right)^{1/2}}{\left(1+\frac{v_+^2}{c^2}\right)^{1/2}}$$

$$\lambda_+' = \lambda_0 \frac{\gamma_+'}{\gamma_0} \quad \text{—— ④.}$$

$$\gamma_0 = \frac{1}{\left[1+\frac{v_0^2}{c^2}\right]^{1/2}} \quad \text{and } \gamma_+' = \frac{1}{\left[+\frac{v'^2}{c^2}\right]^{1/2}} \quad \text{—— ⑤.}$$

⇒ linear charge density of -ve charges in frame s'.

$$\lambda_-' = \lambda_0 \frac{\gamma_-'}{\gamma_0}.$$

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$$\lambda'_+ - \lambda'_- = \frac{\lambda_0}{\gamma_0}\left[\gamma'_+ - \gamma'_-\right] \quad \text{—} ⑦$$

using Eqn. ⑤

$$\lambda'_+ - \lambda'_- = \frac{\lambda_0}{\gamma_0}\left[\frac{1}{\left[1 - \frac{v'^2_+}{c^2}\right]^{1/2}} - \frac{1}{\left[1 - \frac{v'^2_-}{c^2}\right]^{1/2}}\right] \quad \text{—} ⑧$$

Now

$$\left[\frac{1}{\left[1-\frac{v'^2_+}{c^2}\right]^{1/2}} - \frac{1}{\left[1-\frac{v'^2_-}{c^2}\right]^{1/2}}\right] = \frac{1}{\left[1 - \frac{c^4(v_0-v)^2}{c^2(c^2-v_0v)^2}\right]^{1/2}} - \frac{1}{\left[1 - \frac{c^4(v_0+v)^2}{c^2(c^2+v_0v)^2}\right]^{1/2}}$$

$$= \frac{1}{\left[1 - \frac{c^2(v_0-v)^2}{(c^2-v_0v)^2}\right]^{1/2}} - \frac{1}{\left[1 - \frac{c^2(v_0+v)^2}{(c^2+v_0v)^2}\right]^{1/2}} = \frac{c^2 - v_0v - c^2 - v_0v}{\left[(c^2-v_0^2)(c^2-v^2)\right]^{1/2}}$$

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$$= \frac{-2v_0v}{\left[(c^2-v_0^2)(c^2-v^2)\right]^{1/2}} = \frac{-2v_0v}{c^2\left(1-\frac{v_0^2}{c^2}\right)^{1/2}\left(1-\frac{v^2}{c^2}\right)^{1/2}}$$

$$= -\frac{2v_0v\gamma_0\gamma}{c^2}.$$

Substitute this value in Eqn. ⑧, we get net l.c.d

$$\lambda'_+ - \lambda'_- = \frac{\lambda_0}{\gamma_0}\left[-\frac{2v_0v\gamma_0\gamma}{c^2}\right].$$

$$= -\frac{2\lambda_0\gamma v_0 v}{c^2} \quad \text{—} ⑨.$$

Eqn. ⑨ gives the l.c.D of the wire as meas. in $S'$.

Electric field $\vec{E}'$ due to this l.c.D in frame $S'$ is:—

$$E' = \frac{(\lambda'_+ - \lambda'_-)}{2\pi\epsilon_0 x}$$

If $x$ is the ⊥ar dist$^n$ of the charge.

$$E' = \frac{1}{2\pi\epsilon_0 x}\cdot -\frac{2\lambda_0\gamma v_0 v}{c^2}.$$

$$E' = -\frac{2\lambda_0\gamma v_0 v}{2\pi\epsilon_0 x c^2} \quad ⑩.$$

$c = (\text{Velo. of Light}) = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$c^2 = \frac{1}{\mu_0 \varepsilon_0}$ i.e. $\mu_0$ is the permeability of free space and $\varepsilon_0$ is the permittivity of free space.

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Substituting the value of $c^2$ in Eqn (10),

$$E' = -\frac{2\lambda_0 \gamma v_0 v}{2\pi \varepsilon_0 x \cdot \frac{1}{\mu_0 \varepsilon_0}} \Rightarrow E' = -\frac{\mu_0}{4\pi} \frac{4\lambda_0 \gamma v_0 v}{x} \quad (11).$$

Force on charge q as measured in $s'$ is:—

$$F_z' = qE' = \frac{dp_z'}{dt'} = -\frac{\mu_0}{4\pi} \cdot \frac{4q\lambda_0 \gamma v_0 v}{x} \quad (12).$$

$$F_z = \frac{1}{\gamma} F_z' = -\frac{\mu_0}{4\pi} \frac{4q\lambda_0 v_0 v}{x} \quad (13).$$

$(qv = I)$ the current through the wire are measured in s due to the motion of charges,

$$F_z = -\frac{\mu_0}{4\pi} \frac{2I_0 I}{x} \quad (14).$$

⇒ A arrays of charges carry current in the same directⁿ. -ve sign in (14) Eqn shows the +ve array of charges is attracted towards the -ve array of charges.

⇒ $F_z$ is proportional to the product of current,

⇒ " is inversely prop to the distⁿ. x.

CHAPTER-7
Capacitor and properties of Dielectrics :.

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Q1:- What is a Capacitor?

Ans:- Capacitor:- A System Consisting of two Conductors Carring Equal and opposite charges seperated by a certain distn. through a non-(uniform) Conducting medium is Known as Capacitor.

• It is used to store Charge and energy in the form of the electric field.

Q2: Define Capacitance of Capacitor. state and define its SI units?

Ans:- Capacitance of a capacitor is defined as that the ratio of magnitude of charge on the capacitor to the potential difference betn. the Conductors of the Capacitor

$\boxed{C = \frac{Q}{V}.}$ , The SI unit of Capacitance of Conductor is Faraday. (F).

Q3: Define polarization vectore? State its SI unit?

Ans:- Polarization Vector:—> It may be defined as that the ratio of induced dipole moment to the volm. of the subn. is Called polarization vector.

SI unit of P.V :- $Cm^{-2}$.

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Q6: What is a dielectric?

Ans:- A dielectric is an insulating material in which all the Electrons are tightly bounded to the nuclei of the atom. and there are no free Electrons are available for Conduction of Currents. Under the application of External Electric field these materials Can be polarized.

Q7: Define dielectric Constant?

Ans:- It may be defined as that the ratio of original Electric field $E_0$ to the reduced value of the Electric field (E) inside dielectric when it is placed in betn. parallel plates of capacitor are Called Dielectric Constt.

Q8: Why is the dielectric strength of Vaccum is infinite?

Ans:- Dielectric strength of vaccum is infinite bcoz there are no molecule in (the) Vaccum to be polarized.

Q9: An atom having spherically charge distribution is non-polar? why?

Ans:- It is non-polar, bcoz in Case of spherically Symmetric Charge distribution the positive charge centre Coincide With the negative charge centre.

Q10: What is meant by dielectric strength of a dielectric?

Ans:- It is the maximum value of Electric field that a dielectric

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Q4: What is difference betⁿ free charges and bound charges?

Ans:– The electric charges on the plates of the capacitor are called **free charges.** They can move freely. On the other hand, the charges induced on the opposite sides of dielectrics due to polarization. Hence, it is called **bound charges.**

Q5: Distinguish betⁿ polar and Non-polar molecules?

Ans:–

| Polar molecules. | Non-polar molecules. |
| --- | --- |
| (i) The molecules in which the centre of mass of the positive charge does not coincide with the negative charge are called polar molecules | (i) The molecules in which the centre of mass of the positive charge coincides with negative charge are called Non-polar molecules. |
| (ii) They have a net dipole moment. | (ii) They have a zero dipole moment. |
| (iii) Ex. of polar molecules are HCl and $H_2O$ etc. | (iii) Ex. of N.P.M are $CO_2$, $Cl_2$ etc. |

material without breakdown when placed in an Electric field.

Q10: State the factors on which dielectric strength depends?

Ans:— The dielectric strength depends upon the shape of the dielectric material. It also depends upon the medium that surrounds the insulator.

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Q11: Express the Clausius - Mossotti Eqn?

Ans:— Clausius - Mossotti Eqn.:—

$$\boxed{\alpha = \frac{3\varepsilon_0}{N}\left[\frac{K-1}{K+2}\right]}.$$

Q12: Molecules consisting of dissimilar atoms are usually polar. Why?

Ans:— It is due to fact that molecules consisting of dissimilar atoms are usually polar as asymmetric charge distribution.
Ex:- In an Ionic molecule, the Electrons are transferred from one atom to the some other atom in the molecule.

Q13: Define relative permittivity and dielectric constant. What is the relation betn. them?

Ans:— Relative permittivity:— The ratio of the force acting betn. two charges placed in a Vaccum to the force acting betn. the same charges placed at same distn. apart in medium. It is called Relative permitivity.

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• **Dielectric Constant:—** The ratio of original Electric field ($E_0$) to the reduced value of the Electric field is called Dielectric Constant.

The relative permitivity is equal to dielectric Constt. :—

$$\Rightarrow \boxed{\epsilon_r = K}$$

**Q1:- Derive molecular interpretation of Clausius-Mossotti Equation?**

**Ans:—** **Clausius-Mossotti Eqn.:—** Consider a spherical cavity of radius $x$ surrounding a point $p$ inside a polarised dielectric placed in the electric field produced in betn the plates of parallel plate of capacitor. Let $p$ lies at the centre of the cavity. The Electric field at point $p$ is called **molecular Electric field is,**

$$E_m = E_1 + E_2 + E_3 + E_4.$$

• $E_1$ is the External electric field applied when there is no dielectric.

$$E_1 = \frac{\sigma}{\varepsilon_0}.$$

• $E_2$ is the Electric field intensity due to polarization of dielectric. The magnitude of the Electric field is due to:—

$$E_2 = \frac{\sigma_p}{\varepsilon_0}.$$

• $E_3$ is the Electric field intensity due to polarization of the surface of the cavity,

$$\frac{\vec{P}}{n\alpha} = \vec{E} + \frac{\vec{P}}{3\epsilon_0} \quad \text{or} \quad \frac{\vec{P}}{n\alpha} - \frac{\vec{P}}{3\epsilon_0} = \vec{E}.$$

$$\Rightarrow \vec{E} = \left[\frac{1}{n\alpha} - \frac{1}{3\epsilon_0}\right]\vec{P} \quad \text{or} \quad \vec{P} = \frac{\vec{E}}{\left[\frac{1}{n\alpha} - \frac{1}{3\epsilon_0}\right]} \text{ — ③.}$$

We know that $\vec{P} = \epsilon_0 \underset{(\text{Khai})}{\chi} \vec{E} = \epsilon_0 (K-1) \vec{E}$.

Comparing Eqn. ③ and ④

$$\epsilon_0 (K-1) = \frac{1}{\left(\frac{1}{n\alpha} - \frac{1}{3\epsilon_0}\right)}$$

⇒ Solving the above Expression,

$$\boxed{\alpha = \frac{3\epsilon_0 (K-1)}{n(K-2)}}$$

This Eqn. is Known as Clausiu-Mossotti Equation.

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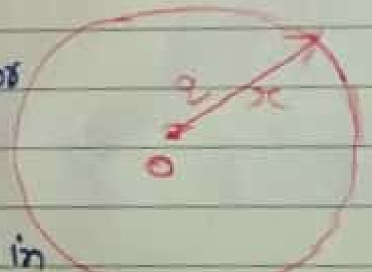
**Q1:- Capacitance of a spherical Conductor:—**



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Consider, an Isolated Spherical Conductor of radius $r$ having charge $q$ on it.

Suppose that, the Cond. is placed in air or Vaccum. The Charge on the spherical Conductor may be defined as that as Considered as point o. The potential at the Surface of the Conductor at any p

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \text{ — ①.}$$

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Capacitance is:—

$$c = \frac{q}{v} \Rightarrow c = \frac{q}{\frac{q}{4\pi\epsilon_0 r}}$$

$$\Rightarrow c = \frac{q}{q} \times 4\pi\epsilon_0 r \Rightarrow \boxed{c = 4\pi\epsilon_0 r}$$

capacitance

Earth of capacitance:-

$$\boxed{c = 4\pi\epsilon_0 r}$$

$$\boxed{c = 711\mu F}$$

## Principle of Capacitance:—

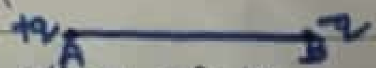
- A capacitance is a device in which used for storing the large amount of Electric charge.

Two metallic Conductor, when one Cond. is Connected to the Earth and other has an ability to store the Electric charge of the Conductor.

★ **Parallel plate Capacitor:—** It is most Commonly used Capacitor. It consist of two Conducting parallel Plates. Its plates are seperated through a certain small distn. due to d seperation betn. the plates. The Electric field on the boundaries of the plate is negligible.

### [Expression for Capacitance of parallel plate Capacitor.]

Consider that +q Charges given to the plate A. Then, Induced -q charges is produced on the left phase of the plate A. Right phase of the plate B



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When, plate B is on Earth, $+q$ charge (charge) produced on the right to the plate B flows to the Earth due to charge $+q$ on plate A.

Electric field is set up bet$^n$. the plates:-

$$\boxed{E = -\frac{dv}{dx}}$$

$$E = \frac{dv}{dx} \text{ (In magnitude)}$$

If, $v$ be the potential difference bet$^n$. two plates and $d$ is the seperation bet$^n$. the plates.

$E = \frac{v}{d}$

$v = Ed$

$E = \frac{\sigma}{\epsilon_0}$

$\sigma = q/A$

$$\boxed{E = \frac{v}{d}} \quad \text{—} ①$$

$$\boxed{v = Ed} \quad \text{—} ②$$

$E = q/\epsilon_0$ $\quad \boxed{E = \frac{q}{A\epsilon_0}}$ i.e $\sigma$ is the surface charge density.

$\Rightarrow v = \frac{q\sigma}{A\epsilon_0}d$ $\quad \Rightarrow c = \frac{qd}{\frac{q}{A\epsilon_0}}$

$c = q/v$

$\Rightarrow \boxed{c = \frac{\epsilon_0 A}{d}}$

$$E = \frac{\sigma}{\epsilon_0} \text{ — } ③ \Rightarrow v = \frac{\sigma d}{\epsilon_0} \quad \left\{ \sigma = \frac{q}{A} \right.$$

$$\Rightarrow \boxed{v = \frac{qd}{A\epsilon_0}}$$

Now, Acc. to the definition of Capacitance:—

$$C = \frac{q}{v}$$

fixed capacitance

variable capacitance

$$C = \frac{q}{\frac{q}{A\epsilon_0}d} \Rightarrow \boxed{C = \frac{\epsilon_0 A}{d}} \text{ Ans.}$$

Q2:- Capacitance of a parallel plate Capacitor when a dielectric slab partially fills the space betn. the plates?

Ans:— Consider a parallel plate Capacitor,

Let A be the area of each plate and d be the Seperation betn. plates.

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$, So plate A is +ve potential and plate B is -ve potential.

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$E_0$ is set up betn. the plates place a dielectric slab of thickness t betn. the two plates.

Electric field betn. plates reduced value.

$$\boxed{E = E_0 - E_p}$$

$$\Rightarrow E = E_0 - \frac{P}{\varepsilon_0}$$

potential diff. betn. two plates of Capacitor.

$$V = Et - E_0(d-t) \quad \text{—①}$$

$$K = \frac{E_0}{E} \Rightarrow E = \frac{E_0}{K} \quad \text{—②}$$

Now, put E in eqn.①,

$$V = \frac{E_0 t}{K} - E_0(d-t)$$

$$\Rightarrow V = E_0\left(d - t + t/_K\right)$$

Acc. to Capacitance

$$C = \frac{q}{V} \Rightarrow C = \frac{q}{\frac{q}{\varepsilon_0 A}\left(d-t+t/_K\right)}$$

$$C = \frac{\epsilon_0 A}{(d - t + t/k)} \Rightarrow C = \frac{\epsilon_0 A}{d - t\left[1 - \frac{1}{k}\right]}$$

Dielectric completely fills the space, $t = d$.

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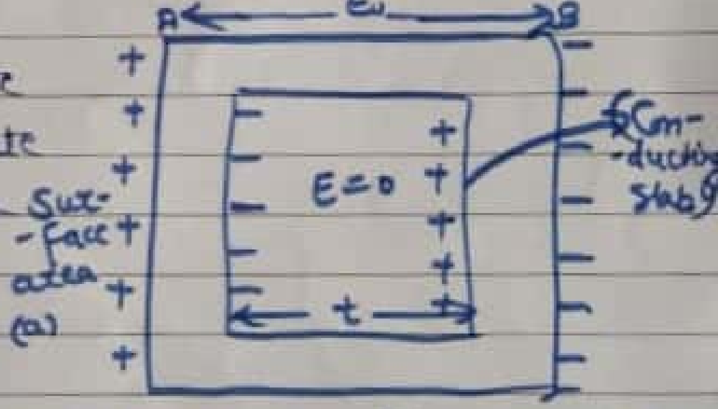
$$\Rightarrow C = \frac{\epsilon_0 A}{d - d\left[1 - \frac{1}{k}\right]} \Rightarrow C = \frac{\epsilon_0 A}{\cancel{d} - \cancel{d} + d/k}$$

$$\Rightarrow C = \frac{\epsilon_0 A}{(d/k)} \Rightarrow C = \frac{\epsilon_0 A k}{d}$$

$$\Rightarrow \boxed{C = C_0 k} \quad \boxed{k > 1}$$

$\boxed{C_0 = \frac{\epsilon_0 A}{d}}$ ⑧ Capacitance of a parallel plate Capacitor, when Conducting slab is introduce betn. the plates of the Conductor.

Ans:— Consider a parallel plate Capacitor having each plate of Area A and d is the separation betn. the plates.




$\boxed{C_0 = \frac{\epsilon_0 A}{d}}$ Suppose that, an Electric field $E_0$ is ~~placed~~ produced betn the plate of the Capacitor and Conducting slab is placed inside the plate of Capacitor.

Potential difference betn. the plates will be:—

$v = Et + E_0(d-t)$.

$E = 0$.

now, $\Rightarrow v = E_0(d-t)$.

$\left(E_0 = \frac{q}{\varepsilon_0 A}\right)$ $\quad v = \frac{q}{\varepsilon_0 A}(d-t)$. $\Rightarrow$ Acc. to the definition of Capacitor,

$C = q/v$.

$\Rightarrow C = \frac{q}{\frac{q}{\varepsilon_0 A}(d-t)} \Rightarrow C = \frac{\varepsilon_0 A}{(d-t)}$

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$\Rightarrow C = \frac{\varepsilon_0 A}{d\left(1-\frac{t}{d}\right)} \Rightarrow C = \frac{C_0}{1-t/d}$.

put $\boxed{t=d}$ $\Rightarrow C = \frac{C_0}{1-d/d} \Rightarrow C = \frac{C_0}{1-1}$.

$\Rightarrow C = \frac{C_0}{0} \Rightarrow \boxed{C = \infty}$. Capacitor is infinite.

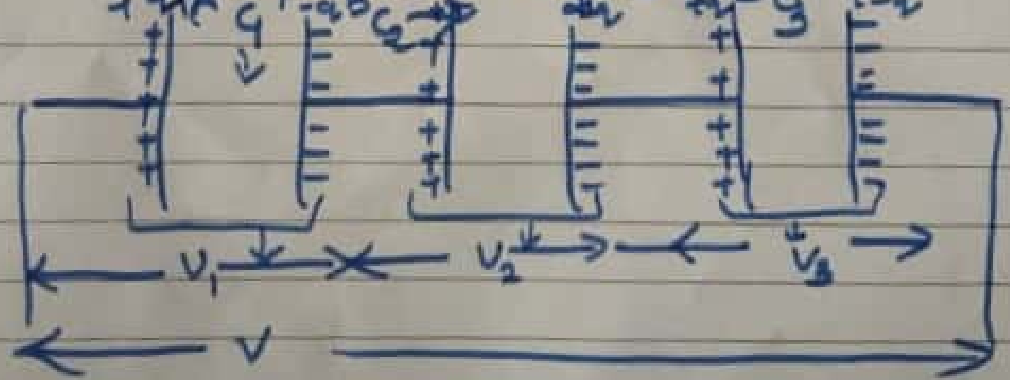
## Grouping of Capacitors:—

(i) In series.

(ii) In parallel.

Capacitors in series:—

$\Rightarrow$



We have to connect three capacitance $C_1, C_2$ and $C_3$.

Now, let $V_1, V_2$ and $V_3$ be the potential diff. across the series combination of three capacitors.

Let $V$ is the total potential difference of three components:—

$\boxed{V = V_1 + V_2 + V_3}$ — ①.

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$\Rightarrow q \propto V$.

$\Rightarrow q = C_1 V_1 \quad \Rightarrow V_1 = \frac{q}{C_1}$ — ②

$\Rightarrow q = C_2 V_2 \quad \Rightarrow V_2 = q/C_2$ — ③

$\Rightarrow q = C_3 V_3 \quad \Rightarrow V_3 = q/C_3$ — ④

Now, put $V_1, V_2$ and $V_3$ in Eqn. ①, we get.

$$\frac{\cancel{q}}{C} = \frac{\cancel{q}}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}.$$

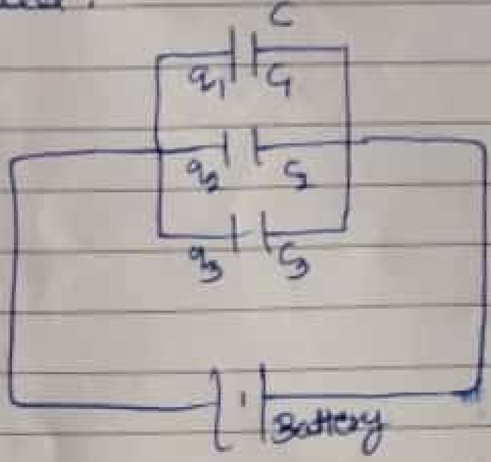
$$\frac{\cancel{q}}{C} = \cancel{q}\left[\frac{1}{C_1} + \frac{1}{C_2} + 1/C_3\right].$$

$$\Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + 1/C_3}$$

Capacitors are connected in series the reciprocal of the ~~the~~ resultant capacitance in series is equal to the sum of the reciprocal of the individual capacitor.

## (ii) Capacitors in parallel:-

Three capacitor of capacitance are connected in parallel.




$\boxed{q \propto V}$ charge is directly proportional to potential diff

$\Rightarrow \boxed{q \propto V} \Rightarrow q = CV$

$\Rightarrow q_1 = C_1 V$ — (2)

$\Rightarrow q_2 = C_2 V$ — (3)

$\Rightarrow q_3 = C_3 V$ — (4).

$\Rightarrow q = q_1 + q_2 + q_3$ — (1)

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$\Rightarrow$ Put Eqn. (2), (3) and (4) in Eqn. (1), we get.

$q = C_1 V + C_2 V + C_3 V$.

$\Rightarrow C\cancel{V} = C_1 V + C_2 V + C_3 V$.

$\Rightarrow \cancel{C_p}\cancel{V} = \cancel{V}(C_1 + C_2 + C_3)$.

$\Rightarrow \boxed{C_p = C_1 + C_2 + C_3}$ Capacitor of capacitance of parallel plate its Equal to sum of the three capacitors $C_1$, $C_2$ and $C_3$. Ans.

# CHAPTER-7

* Molecular field in a dielectric (Clausius–Mossotti Eqn).

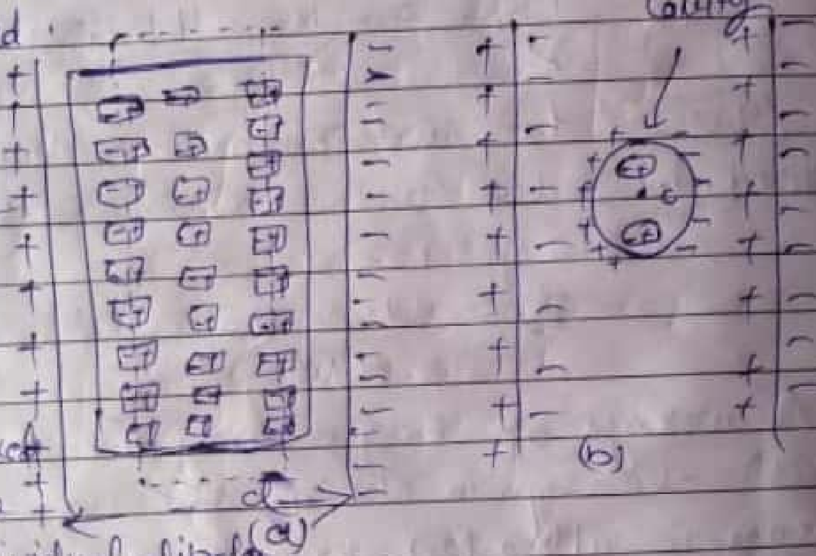
⇒ The electric field $\vec{E}$ which polarises a molecule of the dielectric is known as molecular field. It is denoted by $\vec{E_m}$. 146

This electric field is not equal to the external electric field intensity which is given by $E = \frac{V}{d}$, V is the P.D bet^n equally and oppositely charge plates molecules in the dielectric.

[redacted]

* Molecular field can be calculated. Consider a dielectric slab in bet^n the plates of ‖ capacitor.

(a) The dielectric is polarized and its opposite faces acquire equal and opposite charges. Now cut a spherical cavity of radius x surr. a point in which the M.F can be calculated.



The cavity is put in its original position in the dielectric. The molecules which are replaced by cavity are treated as individual dipole.

Electric field intensity at the centre (C) of the cavity is:—

$$E_m = E_1 + E_2 + E_3 + E_4 \quad \text{---} \ (1)$$

$E_1 = \sigma/\epsilon_0$, is the the electric field intensity bet^n the plates of capacitor with no dielectric in it.

$\sigma$ = surface charge density of the charge on the plates of the capacitor.

$E_2 = -\frac{\sigma_p}{\epsilon_0}$ is the electric field intensity due to the polarised charged on the opposite faces of the dielectric.

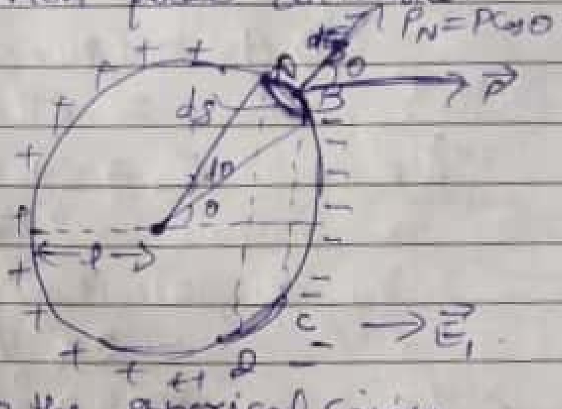
$\sigma_p$ = Surface charge density due to polarized charges on the opposite faces of the dielectric.



$E_3$ is the electric field intensity due to polarised charges on the surface of the cavity.



$E_4$ is the electric field intensity due to dipoles inside the cavity which is zero for non-polar dielectrics.




Eqn. (i) becomes:—

$$E_m = \frac{\sigma_i}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0} + E_3 \quad \text{—(2)}$$

Consider a small element area $ds$ and angular width $d\theta$ at an angle $\theta$ with the direcn of $E_1$ on the spherical cavity.

Let $\vec{P}$ be the polarisation vector at an element of area $ds$. The component of $\vec{P}$ normal to the surface of the element is:—

$$\boxed{P_N = P\cos\theta}$$

The ~~small~~ charge on the small element:—

$$dq = (P\cos\theta)\,ds$$

$\Rightarrow$ Electric field intensity ~~at the~~ centre of cavity due to this charge.

$$dE_3 = \frac{1}{4\pi\epsilon_0}\frac{dq}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{(P\cos\theta)ds}{x^2} \quad \text{—(3)}$$

Resolve $dE_3$ into two components (i) $dE_3\cos\theta$ along the polarisation vector (ii) $dE_3\sin\theta$ ⊥lar to polarisation vector.

$dE_3\sin\theta$ components of upper and lower halves of the cavity cancel each other, $dE_3\cos\theta$ comp. gets added up.

Net Electric field intensity at the centre of the cavity due to the charges on the cavity is:—

$$\int dE_3 = \int_0^{\pi} dE_3 \cos\theta$$

$$E_3 = \int_0^{\pi} \frac{1}{4\pi\varepsilon_0} \frac{P(\cos\theta)\, ds}{x^2} \cos\theta$$

$$E_3 = \frac{1}{4\pi\varepsilon_0} \frac{P}{x^2} \int_0^{\pi} \cos^2\theta\, ds$$

(area of the ring part)

$$\Rightarrow ds = (2\pi x \sin\theta)\, x\, d\theta$$
$$= 2\pi x^2 \sin\theta\, d\theta$$

$$E_3 = \frac{1}{4\pi\varepsilon_0} \frac{P}{x^2} \int_0^{\pi} 2\pi x^2 \sin\theta \cos^2\theta\, d\theta$$

$$\frac{1}{4\pi\varepsilon_0} P \cdot 2\pi x^2 \qquad E_3 = \frac{P}{2\varepsilon_0} \int_0^{\pi} \sin\theta \cos^2\theta\, d\theta$$

$$\frac{P}{2\varepsilon_0} \qquad = \frac{P}{2\varepsilon_0} \times \frac{2}{3} = \frac{P}{3\varepsilon_0} \qquad \left[\int_0^{\pi} \sin\theta \cos^2\theta\, d\theta = \frac{2}{3}\right]$$

Evaluation of $\int_0^{\pi} \sin\theta \cos^2\theta\, d\theta$

Put $x = \cos\theta$, $dx = -\sin\theta\, d\theta$

Lt change from $\theta = 0$ to $x = 1$ and from $\theta = \pi$ to $x = -1$.

$$\int_0^{\pi} \sin\theta \cos^2\theta\, d\theta = \int_1^{-1} -x^2\, dx = \int_{-1}^{1} x^2\, dx$$

$$= 2\int_0^1 x^2\, dx = 2\left[\frac{x^3}{3}\right]_0^1 = \frac{2}{3}$$

Put this value in Eqn. (2)

$$E_m = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_P}{\varepsilon_0} + \frac{P}{3\varepsilon_0} \quad \text{--- (3)}$$

$$E_m = E + \frac{P}{3\varepsilon_0} \quad \text{--- (4)}$$

$E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_P}{\varepsilon_0}$ is the net electric field intensity betn. the plates of the capacitor on the ind. of diele. slab

Electric field is known as polarizability ($\alpha$)



$$\alpha = \frac{p}{E_m}$$

$$\vec{p} = \alpha \vec{E_m} \quad \text{—} ⑥$$

If n no. of molecules per unit volm. then $\vec{P}$ is:—

$$\vec{P} = n\vec{p}$$

$$\vec{P} = n\alpha\vec{E_m}$$

$$\alpha = \frac{p}{E_m}$$

$$\Rightarrow \boxed{\vec{E_m} = \frac{\vec{P}}{n\alpha}} \quad \text{—} ⑦$$

put this value in Eqn. ⑤

$$\frac{\vec{P}}{n\alpha} = \left[\vec{E} + \frac{\vec{P}}{3\varepsilon_0}\right]$$

$$\vec{P} = n\alpha\left[\vec{E} + \frac{\vec{P}}{3\varepsilon_0}\right] \quad \text{—} ⑧$$

$K = 1 + \frac{P}{\varepsilon_0 E}$, K is dielectric Const.

$$\vec{P} = (K-1)\varepsilon_0\vec{E}$$

$$\text{or } \boxed{\vec{E} = \frac{\vec{P}}{(K-1)\varepsilon_0}}$$

Put this value in Eqn. ⑧

$$\vec{P} = n\alpha\left[\frac{\vec{P}}{(K-1)\varepsilon_0} + \frac{\vec{P}}{3\varepsilon_0}\right]$$

$$1 = n\alpha\left[\frac{1}{(K-1)\varepsilon_0} + \frac{1}{3\varepsilon_0}\right]$$

$3\varepsilon_0 (K-1)\varepsilon_0$

$= 3\varepsilon_0 + \varepsilon_0 K - \varepsilon_0$

$= \varepsilon_0 (3 + (K-1))$

$$= n\alpha\left[\frac{3+K-1}{3(K-1)\varepsilon_0}\right]$$

$$= n\alpha\left[\frac{(K+2)}{3(K-1)\varepsilon_0}\right]$$

$$\boxed{\alpha = \frac{3\varepsilon_0(K-1)}{n(K+2)}} \quad \text{—} ⑨$$

Eqn ⑨ is known as Claussius-Mossotti Eqn.

Dipole moment per unit of a molecule per unit polarising Electric field is known as polarizability ($\alpha$)

# CHAPTER-9
## Magnetic Fields in Matter.

(i) Ferromagnetic Substance:— These are those Subⁿ which are strongly attracted by the magnets. Eg: are Fe, Ni, Co, Steel etc. These Subⁿ get magnetized in the directⁿ of applied magnetic field, called Ferromagnetic Subⁿ.

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(ii) Paramagnetic Substance:— These are those subⁿ which are feebly attracted by magnets. Eg:- Al and Na etc. So, These subⁿ gets magnetized in the directⁿ of applied magnetic field.

(iii) Diamagnetic Substance:— These are those subⁿ which are feebly repelled by the magnets. Eg:- Cu, Bi, Pd etc. These substances gets magnetized in a directⁿ opposite to the directⁿ of applied magnetic field.

Q1:- What are Non- Magnetic Substance?

Ans:— Non - Magnetic Substance:— There are certain atoms in which the electrons are completely paired, for each electron spinning in one direction, on the another electron spinning in opposite directⁿ Then, the resultant of magnetic moment dipole is Zero. They are called Non- Magnetic Substance.

• Vaccum is the only non - Magnetic Subst

Q2:- What do you mean by atomic dipole?

Ans:— Atomic dipole :— The moving Electrons around the nucleus call constituents a current loop and hence behaves like a dipole called Atomic dipole.

Q3: What is Bohr magneton? calculate its value.

Ans:— Magnetic moment of the orbiting Electron is

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$$\mu_m = n\left[\frac{eh}{4\pi m_e}\right]$$

⇒ Bohr magneton is equal to the magnetic dipole moment of an atom in which electron revolves around the nucleus.

Q4: what is gyromagnetic orbital?

Ans:— The ratio of orbital magnetic moment dipole to the angular momentum of the orbiting Electron, are called gyromagnetic orbital.

Q5: What is magnetisation?

Ans:— When a magnetic subn. is placed in magnetic field. Then, the atoms and molecules get aligned in the direction of applied field. The substance gets magnetised and phenomenon are called magnetisation

$i.e. \vec{M} = -\mu_m$

Q6: What is magnetization Vector?

Ans:— It is the ratio of magnetic dipole moment to the Vol$^m$. is called magnetization Vector.

Q7: How can a magnet be demagnetised Completely?

Ans:— A magnet is demagnetised Completely if the atomic dipoles are randomly oriented. The

156 alignment of dipoles are broken and thermal energy are increases. Magnet can be magnetized Completely by heating it.

Q8: Magnetic behaviour of magnetic substance dec with increasing temp. Explain.

Ans:— Magnetic behaviour of magnetic Sub$^n$. is due to alignment of magnetic dipoles in a particular direct$^n$. The alignment is disturbed by the thermal Energy. when temp. increases, the magnetic dipole moment disturbed and the Sub$^n$. losses the magnetic properties.

Q9. Why does a paramagnetic substance move from weaker to stronger part of non-uniform magnetic field?

Ans:— Paramagnetic sub$^n$. are feebly attract by the magnet. It Experiences a more force in the stronger field to the weaker field.

Q10: Compare Soft iron and hard steel as magnetic material?

Ans:- (i) Susceptibility:- The susceptibility of soft iron is more than that of hard steel.

(ii) Permeability:- The permeability of soft iron is more than that of Hard steel.

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(iii) Retentivity:- The retentivity of soft iron is more than that of hard steel.

(iv) Hysteresis loss:- The area of hysteresis loss per unit vol$^m$ of Soft iron is less than that of hard steel.

(v) Coercivity:- The Coercivity of Soft iron is less than that of hard steel.

Q11 What type of Substance would you prefer for making a permanent magnets?

Ans:- (i) High Retentivity.
(ii) High Coercivity

Q12: What would be prefer while consideration for making Electromagnets?

Ans:- (i) High permeability.
(ii) Low coercivity
(iii) Low Retentivity
(iv) Low Hysteresis loss.

Q13 What would be Consideration for making core of transformers?

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Ans:- (i) High permeability
(ii) Low Hysteresis.

Q14: What are permalloys?

Ans:- These are Some alloys of iron and nickel which have high permeability bec. of which they are widely used in the cores of transformers and chockey.

Q15 Why are steel and alnico used for making perma-nent magnets?

Ans:- They are used for making permanent magnet bec. they have very large Value of Retentivity of Coercivity.

Q16: What is Curie temp?

Ans:- The temp at which the ferromagnetic substance becomes paramagnetic substance called Curie temp.

Q17: Why an ordinary iron piece does not behave as a magnet?

Ans:- An Ordinary iron piece has large no. of atoms Each atoms behaves a magnetic dipole. The magnetic dipole is distributed in the iron piece in the absence of any external magnetic field

159 The net magnetic dipole moment of the iron piece is zero. An ordinary iron piece does not behave as a magnet.

Q18: Why is ferromagnetism not found in liquid and gases?

Ans:- Ferromagnetic occurs when many atoms is anisotropically align into its domain and we get to give a magnetic dipole moment in solids. The atoms of liquid and gases do not align in same direction due to thermal fluctuations and ferromagnetism is not found in liquids and gases.

**Magnetic field Intensity (H) :—**

⇒ The current flowing through the conductor and which can be measured with ammeter are called free currents. The source of these currents may have battery and other source of emf in the circuit. The current which are associated with molecular and atomic dipole are called bound currents. Magnetization vector are also called bound currents.

For steady currents, the Ampere's circuital law is,

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad \text{--- (1)}$$

⇒ The total current density is the sum of current densities of free and bound charges of bound charges.

$$\vec{J} = \vec{J}_f + \vec{J}_m \quad \text{--- ②}$$

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Using Eqn. ①, we have,

$$\Rightarrow \vec{\nabla} \times \vec{B} = \mu_0 (\vec{J}_f + \vec{J}_m) \quad \text{--- ③}$$

$$\Rightarrow \boxed{\vec{J}_m = \vec{\nabla} \times \vec{M}}$$

- Now, put $\vec{J}_m$ in Eqn. ③ we get

$$\vec{\nabla} \times \vec{B} = \mu_0 \left[\vec{J}_f + (\vec{\nabla} \times \vec{M})\right]$$

$$\Rightarrow \vec{\nabla} \times \vec{B} = \left[\mu_0 \vec{J}_f + \mu_0 (\vec{\nabla} \times \vec{M})\right]$$

$$\Rightarrow \vec{\nabla} \times \vec{B} - \mu_0 (\vec{\nabla} \times \vec{M}) = \mu_0 \vec{J}_f$$

$$\Rightarrow \vec{\nabla} \left[\vec{B} - \mu_0 (\vec{M})\right] = \mu_0 \vec{J}_f$$

$\Rightarrow$ Dividing by $\mu_0$ on both sides, we get,

$$\Rightarrow \vec{\nabla} \left[\frac{\vec{B}}{\mu_0} - \vec{M}\right] = \vec{J}_f \qquad \left\{ \left(\frac{\vec{B}}{\mu_0} - M = \vec{H}\right) \right.$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{H} = \vec{J}_f}$$ which is called differential form of Ampere Law.

Integrating Eqn. ③ over a surface, we get.

$$\Longrightarrow \iint_S \vec{\nabla} \times \vec{H} \cdot d\vec{s} = \iint_S \vec{J}_f \cdot d\vec{s}.$$

Acc. to Stoke's theorem,

$$\Longrightarrow \iint_S \vec{\nabla} \times \vec{H} \cdot d\vec{s} = \oint_C \vec{H} \cdot d\vec{l}$$

$$\oint_C \vec{H}\cdot d\vec{l} = \iint_S \vec{J}_f \cdot d\vec{s}$$

161 $$\boxed{\oint_C \vec{H}\cdot d\vec{l} = I_f}$$

⇒ which is known as Integral form of Ampere's law for magnetic materials.

SI unit of Magnetic field Intensity :– $Am^{-1}$.

- Magnetic Induction :– (B) Magnetic Material is placed in current carrying solenoid, then the material gets magnetized and sum of total magnetic field is given by,

$$B = B_0 + B_m \quad \text{—— (1)}$$

where, ~~$B_0$ is the ma~~ $B_0 = \mu_0 nI$ is the applied magnetic field due to current carrying solenoid. On the other hand $B_m$ is the magnetic field due to magnetism.

So, $B_m = \mu_0 M$ — (2)

Now, $B_0$ and $B_m$ put in eqn. (1), we get:

$$\Rightarrow B = \mu_0 nI + \mu_0 M.$$

$\{ nI = H$. magnetic field intensity

$$\Rightarrow B = \mu_0 H + \mu_0 M.$$

$$\Rightarrow B = \mu_0 (H + M).$$

⇒ which is called magnetic Induction ($\vec{B}$)

: SI unit of Magnetic Induction is Tesla (T)

- **Magnetic permeability:—** It is denoted by $\mu$. Magnetic permeability of the material is the measure of degree or extent to which magnetic field can penetrate the material.

162 It has been found Experimentally that the magnetic Induction $\vec{B}$ is directly proportional to the Magnetic field Intensity.

$$\vec{B} \propto \vec{H}.$$

$$\Rightarrow \vec{B} = \mu \vec{H}.$$

Where, $\mu$ is the magnetic permeability.

Now, $\boxed{\mu = \frac{B}{H}}$. SI unit of magnetic permeability is $Hm^{-1}$.

- **Magnetic susceptibility ($\chi_m$):—**

Magnetic Susceptibility of the material is the measure of Ease with which the material gets magnetised. Magnetic susceptibility indicates that the degree of Magnetization of the (magnetic (field) material in response to the magnetic field.

Now, Magnetization Vector ($\vec{M}$) is directly proportional to the Magnetic field intensity $\vec{H}$.

$$\Rightarrow \vec{M} \propto \vec{H}.$$

$$\Rightarrow \vec{M} = \chi_m \vec{H}.$$

$$\Rightarrow \boxed{\chi_m = \frac{M}{H}}.$$

Since; M and H have the dimensions. But, $\chi_m$ has dimensionless quantity.

## ☆ Relation Betn. Magnetic Permeability and Magnetic Susceptibility

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$$\boxed{B = \mu_0 (H + M)} \quad \text{--- (1)}$$

⇒ Dividing b.s by H.

$$\Rightarrow \frac{B}{H} = \mu_0 \left(1 + \frac{M}{H}\right) \quad \text{--- (2)}$$

$$\Rightarrow \frac{B}{H} = \mu, \quad \frac{M}{H} = \chi_m \quad \text{--- (3)}$$

Now, ~~put in eqn (2) put in (1)~~
Eqn. (3) is put in (2) eqn. we get.

$$\Rightarrow \mu = \mu_0 (1 + \chi_m).$$

$$\Rightarrow \frac{\mu}{\mu_0} = (1 + \chi_m) \qquad \left\{ \frac{\mu}{\mu_0} = \mu_r \right.$$

$$\Rightarrow \boxed{\mu_r = (1 + \chi_m)}$$

i.e $\frac{\mu}{\mu_0} = \mu_r$ is the relative permeability of the material. It is dimensionless quantity. Ans.

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## Properties of Dia, Para and Ferromagnetic Substance :-

164 Properties of diamagnetic substance :-

(i) They are repelled by magnet.

(ii) A diamagnetic substance moves from stronger to weaker part of the magnetic field.

(iii) A dia. subn. is placed in magnetic field, the lines of force do not prefer of the magnetic field.

(iv) A dia. subn. the permeability of this subn. is less than one $\mu < 1$.

(v) A dia. subn. is weakly magnetized in direcn opposite to the direction of magnetic field.

(vi) The susceptibility of dia. subn, is -ve.

(vii) The dia. subn. does not obey Curie law.

## Properties of paramagnetic substance :-

(i) They are feebly attract the magnet.

(ii) A para. subn. moves from weaker part to the stronger part of the magnetic field.

(iii) A para. sub$^n$. is placed in magnetic field, then the lines of force prefer to pass through it.

(iv) The permeability of para. sub$^n$. is greater than 1. $\mu > 1$.

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(v) The para. sub$^n$. gets magnetised weakly in the direct$^n$ of applied magnetic field.

(vi) The susceptibility of para. sub$^n$. is +ve.

(vii) The paramagnetic sub$^n$. obey Curie's law

Properties of ferromagnetic sub$^n$:—

(i) They are strongly attract the magnet

(ii) A ferromagnetic sub$^n$. moves from weaker part to the stronger part of the magnetic field.

(iii) A ferromagnetic sub$^n$. is placed in magnetic field, the lines of force prefer to pass through it.

(iv) The permeability of ferr. sub$^n$ is larger than 1 $(\mu >> 1)$.

(v) The ferro sub$^n$. gets strongly magnetized in the direct$^n$. of applied magnetic field.

(vi) The susceptibility of ferro. sub$^n$ is +ve.

# Langevin's theory of Paramagnetism



The orbital and spin motion of an electrons of nuclear ~~of an atom, which give rise to dipole moment.~~

A subⁿ is paramagnetic, if it has net dipole moment. But the atoms in the materials are randomly distributed, dipole moments are also randomly distributed. Net dipole moment is zero.

When an External magnetic field is applied, the dipoles start aligning in the directⁿ of applied field. But the thermal motion of an atom does not allow the complete alignment. If the strength of M.F is increased, temp. is dec. The alignment of dipole moment tends to be complete and material is said to be magnetised.

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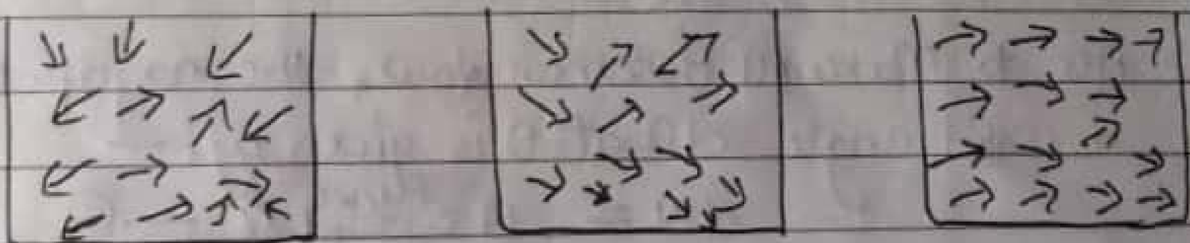
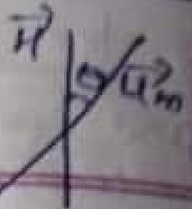


Without magnetic field. (a)    Weak magnetic field (b)    In strong Magnetic field. (c)

Langevin Explained the paramagnetic behaviour of kinetic theory of gases of the dipoles.

Consider unit volᵐ of paramag. Sample having $N$ is the no. of atoms.

Torque acting on the dipoles $\vec{T} = \vec{\mu_m} \times \vec{H}$, $\vec{\mu_m}$ is the magnetic dipole moment of each dipole and $\vec{H}$ be the magnetic field intensity.




$\tau = \mu_m H \sin\theta$

Small amount of work done by external agency displacing the dipole through an angle $d\theta$.

$$d\omega = \mu_m H \sin\theta \, d\theta .$$

Total work done in displacing the dipole from $\theta = 0$ and $\theta = \theta$ is :—

$$\omega = \int d\omega = \int_0^\theta \mu_m H \sin\theta \, d\theta = -\mu_m H \cos\theta .$$

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$$\omega = -\vec{\mu}_m \cdot \vec{H}$$

This work done is stored in magnetic potential energy $U$.

$$U = -\vec{\mu}_m \cdot \vec{H} \quad \text{—— (1)}$$

Acc. to Maxwell Boltzman law, the no. of dipoles within a solid angle $d\Omega$ at $\theta$ is given by :—

$$dN = A e^{-U/kT} d\Omega \quad \text{—— (2)}$$

$A$ is a const. of prop. and $K$ is Boltzman const.

$$d\Omega = \frac{\text{area of ring bet}^n\ \theta \text{ and } d\theta}{R^2} = \frac{2\pi R \sin\theta \, R \, d\theta}{R^2}$$

$$d\Omega = 2\pi \sin\theta \, d\theta .$$

$$dN = A \times 2\pi \sin\theta \, d\theta \, e^{-U/kT}$$

$$dN = C e^{-\vec{\mu}_m \cdot \vec{H}/kT} \cdot \sin\theta \, d\theta .$$

$c = 2\pi A$ is Const.

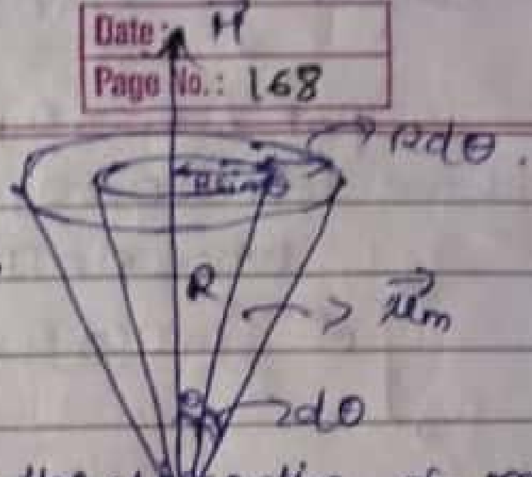
$dN = c\, e^{\mu_m H \cos\theta / KT} \sin\theta\, d\theta$

$dN = c\, e^{\alpha\cos\theta} \sin\theta\, d\theta$

$dN = c\, e^{\alpha\cos\theta} \sin\theta\, d\theta$ — ③.

$\Rightarrow \alpha = \dfrac{\mu_m H}{KT}$ — ④.






Component of dipole moment in the direction of magnetising field $\mu_m \cos\theta$,

Total dipole moment per unit Vol$^m$ is

$\Rightarrow dp_m = dN\, \mu_m \cos\theta$.

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$= c\, e^{\alpha\cos\theta} \sin\theta\, d\theta\, \mu_m \cos\theta$.

$= c\, e^{\alpha\cos\theta} \mu_m \cos\theta \sin\theta\, d\theta$

Total dipole moment of all dipoles per unit vol$^m$ is:—

$\Rightarrow P_m = \int_0^{\pi} c\, e^{\alpha\cos\theta} \mu_m \sin\theta \cos\theta\, d\theta$ — ⑤

Total no. of atom in sample:—

$\Rightarrow N = \int_0^{\pi} dN = \int_0^{\pi} c\, e^{\alpha\cos\theta} \sin\theta\, d\theta$ — ⑥.

Average dipole moment per unit atom in the direct of Magnetic field is:—

$\bar{\mu}_m = \dfrac{P_m}{N} = \dfrac{c\mu_m \int_0^{\pi} e^{\alpha\cos\theta} \sin\theta \cos\theta\, d\theta}{c \int_0^{\pi} e^{\alpha\cos\theta} \sin\theta\, d\theta}$ — ⑦.

Date:
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Consider $\int_0^{\pi} e^{\alpha \cos\theta} \sin\theta \cos\theta \, d\theta$ ?

Put $x = \cos\theta$, $dx = -\sin\theta \, d\theta$.

When $\theta = 0$, $\cos 0 = 1$.

When $\theta = \pi$, $x = \cos\pi = -1$.

$$\int_0^{\pi} e^{\alpha \cos\theta} \sin\theta \cos\theta \, d\theta = -\int_1^{-1} e^{\alpha x} x \, dx$$

$$= \int_{-1}^{+1} x \cdot e^{\alpha x} \, dx = \int_{-1}^{1} x \cdot e^{\frac{\mu_m H x}{kT}} dx \quad \text{— (8)}$$

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At very high temp. :—

$$\frac{\mu_m H}{kT} << 1.$$

$$e^{\mu_m H x / kT} = \left[1 + \frac{\mu_m H x}{kT}\right] \text{— (9)}$$

Eqn (8) becomes :—

$$\int_0^{\pi} e^{\alpha \cos\theta} \sin\theta \cos\theta \, d\theta = \int_{-1}^{1} x \left[1 + \frac{\mu_m H x}{kT}\right] dx$$

$$= \left[\int_{-1}^{1} x \, dx + \int_{-1}^{1} \frac{\mu_m H}{kT} x^2 \, dx\right].$$

$$= \left[\left\{\frac{x^2}{2}\right\}_{-1}^{1} + \frac{\mu_m H}{kT}\left\{\frac{x^3}{3}\right\}_{-1}^{1}\right].$$

$$= \left[0 + \frac{2\mu_m H}{3kT}\right] = \left[\frac{2}{3}\frac{\mu_m H}{3kT}\right]$$

Also $\int_0^{\pi} e^{\alpha \cos\theta} \sin\theta \, d\theta$

$$= -\int_1^{-1} e^{\alpha x} dx = \int_{-1}^{1} e^{\alpha x} dx.$$

$$= \int_{-1}^{1} e^{\frac{\mu_m H x}{KT}} dx.$$

Eqn (a) we get.

$$\int_0^{\pi} e^{\alpha \cos\theta} \sin\theta \, d\theta = \int_{-1}^{1} \left[1 + \frac{\mu_m H x}{KT}\right] dx.$$

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$$= \left[(x)_{-1}^{1} + \frac{\mu_m H}{KT}\left[\frac{x^2}{2}\right]_{-1}^{1}\right].$$

$$= \left[1 + 1 + 0\right] = 2 \quad \text{— (11)}.$$

Using Eqn. (10) and (11) in (9) Eqn. we get

$$\bar{\mu}_m = \frac{\mu_m \times 2/3 \; \mu_m H/KT}{2} = \frac{\mu_m^2 H}{3KT}.$$

Magnetising produced in the sample is:—

$$M = N \bar{\mu}_m$$

$$\boxed{M = N \frac{\mu_m^2 H}{3KT}}$$

$$M = \chi_m H \quad \text{or} \quad \vec{M} = \chi \vec{H}.$$

$\chi_m = \frac{N \mu_m^2}{3KT}$ is the magnetic susceptibility for paramagnetic substance.

$\Rightarrow \chi_m \propto \frac{1}{T}$, This is called Curie law. Paramagnetic substance obey Curie law.

$= \cos y . x^2 \sin y \cos y (\cos^2 z + \sin^2 z) + x \sin y . x \sin^2 y (\cos^2 z + \sin^2 z)$

$= x^2 \sin y \cos^2 y + x^2 \sin^3 y = x^2 \sin y (\cos^2 y + \sin^2 y)$

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$= x^2 \sin y$ Ans.

Physics:-

*⇒ Ferromagnetic substance:- are those subn. which are strongly attracted by the magnet are called ferromagnetic subn. These subn. gets strongly magnetised in the directn. of magnetic field.

Example are:- Fe, Ni, Co and Steel, etc.

⇒ Antiferromagnetism:- are those subn. which have no External magnetic moment, They are not attracted in the macroscopic sense.

Now,

* The main diff. betn. ferromagnetism and Anti-ferromagnetism is the alignment of parallel magnetic moment. Ferromagnetic subn./materials have magnetic moments that a align parallel to the applied magnetic field and antiferromagnetic material have antiparallel magnetic moments.

Antiferromagnetism:- Ex: Hematite, Cr, alloys and FeMn.

⇒ Paramagnetic Substance:- are those subn. which are feebly attracted by the magnets in applied magnetic field. These subn.s get feebly/weakly magnetised in the directn. of applied magnetic field.

Ex:- Al, Na, $NiSO_4$ etc.

⇒ Diamagnetic Sub/Material :— are those material which are feebly repelled by the magnet are called ~~Diamagnetism materials. These sub. get~~ weakly ~~or~~ feebly magnetized in a direct^n opposite to the direct^n of applied magnetic field.

Ex. :— Cu, Bi, Pb, black etc.

** Hysteresis :— The lagging of intensity of magnetisation (M) behind the magnetising field H, when a ferromagnetic sub^n is taken through a cycle of magnetisation and demagnetisation is called Hysteresis.

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Advantage of Hesteresis :—

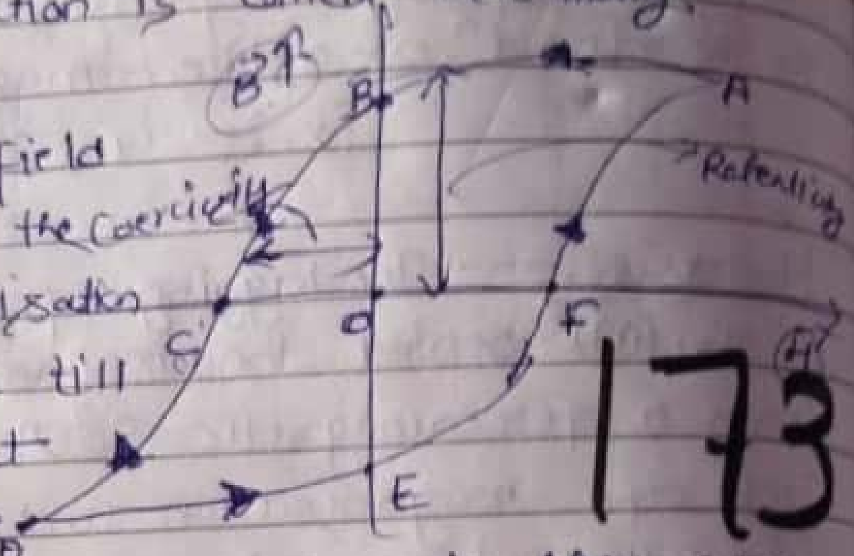
The significance of Hysteresis loop is that it provides information such as retentivity, coercivity, susceptibility and energy loss during one cycle of magnetisation for each ferromagnetic material.

* Hysteresis loop :— Consider an unmagnetized bar of a ferromagnetic sub^n. It is placed in a magnetising field H whose strength can be changed. If H is increased and bar is ~~slowly~~ magnetised. The intensity of magnetisation and M and magnetising ~~field H increase with inc. in~~ Magnetic Induction B inc. with inc. in magnetising field H.

H & magnetic field

If (H) Magnetising field is slowly decreased and B also decreases. Magnetising field is made is made zero, the intensity of magnetic field is not zero, but equal to OB.

~~This intensity of~~ $(\vec{m})$ magnetisation ~~of~~ $(\vec{B})$ Magnetic Induction is called Retentivity.




If $\vec{H}$ magnetising field is reversed, then the intensity of magnetisation $(\vec{m})$ reduces along BC till it becomes to zero at point C. To reduce the retentivity to zero, we have to apply a magne-tising field H equal to OC, in the reverse direcⁿ.

The magnetising field require to make retentivity zero is called Coercivity.

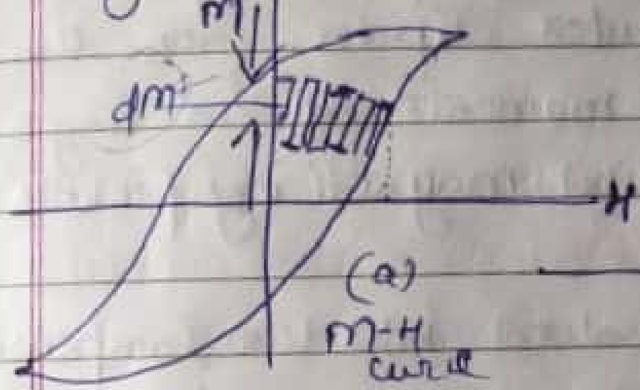
If the reverse magnetising field H is further increased the $\vec{m}$ is further change in CD. The point D represents the saturation (value) of the material. On dec. the field again in steps and then increasing in reverse direcⁿ The curve follows the DEFA. The closed curve is ABCDEFA is called hysteresis curve/loop. The Hysteresis word means "a lagging behind".

* The lagging of Intensity of magnetisation $(\vec{m})$ behind the magnetising field H per cycle ~~of magnetisation~~ when a ferromagnetic subⁿ is taken through a cycle of magnetisation and demagnetisation are called Hysteresis loop.

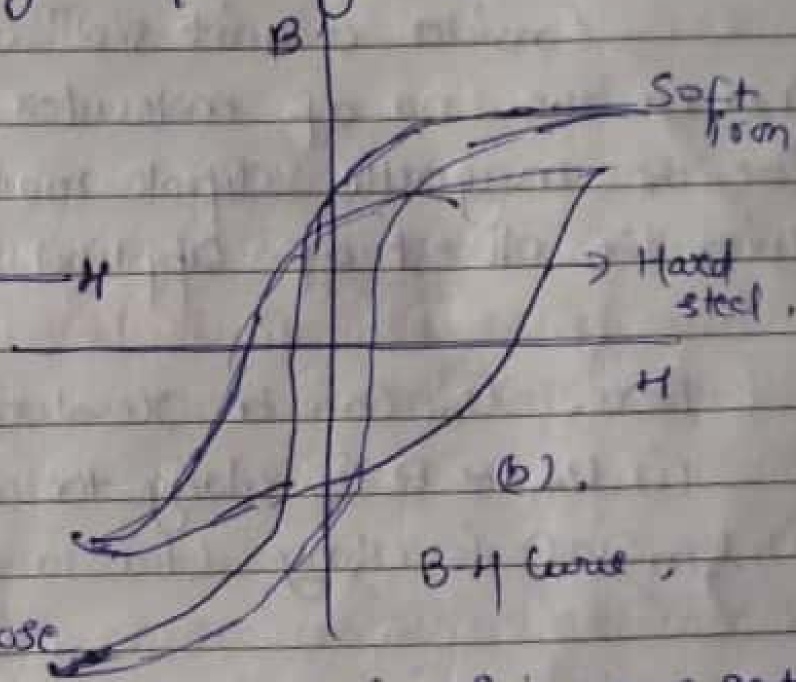
M-H Curve

The M-H curve is identical to B-H curve with the only diff. in case of M-H. we get a saturation stage when the curve is straight and parallel to the abscissa, B-H curve, the saturation is not parallel to abscissa.

Diff. materials have different hysteresis loop. So, Diff. betn Soft iron and Hard steel. It may be noted that, the area of loop represents the loss of energy per cycle of magnetisation or demagnetization.




(a) M-H curve

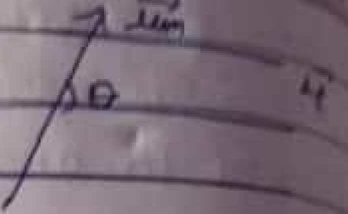



(b). B-H curve.

To make permanent magnets, we use those subn which have large value of coercivity and retentivity. Hard steel, Cobalt steel (contain Cobalt, tungsten carbon and iron) are the ex. of such subn. Now a days alloy alnico [Iron (54%), Nickel (18%) Cobalt (12%), Aluminium (10%) and copper (6%)] is widely used to make permanent magnets.

## Loss of Energy Due to Hysteresis

⇒ When a ferromagnetic Subⁿ. is taken ~~through a magnetisation and dema~~-gnetisation. There are always occur some loss of Energy, bcz, work done on the specimen during magnetisation are not recovered completely on demagnetisation. A net loss of energy takes place through a Hysteresis cycle is called Hysteresis loss.



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Consider a unit volᵐ. of ferromagnetic Subⁿ. & n be the no. of molecular dipoles. Then $\vec{\mu}_m$ be the magnetic dipole moment at an angle θ with the direct of applied magnetising field ($\vec{H}$).

Here, $\vec{\mu}_m$ can be resolved into two components:-

(i) $\mu_m \cos\theta$ is along the directⁿ of $\vec{H}$ and

(ii) $\mu_m \sin\theta$ is along ⊥lar to the directⁿ of $\vec{H}$.

There is no magnetisation ⊥lar to $\vec{H}$, Sum of all components of $\vec{\mu}_m$ ⊥lar to $\vec{H}$ is zero.

$$\Sigma \mu_m \sin\theta = 0.$$

⇒ Specimen acquire magnetisation M in the direct of H

$$M = \Sigma \mu_m \cos\theta$$

$dM = -\sum \mu_m \sin\theta d\theta$ —— ①.

Torque acting on the dipole is ~~placed in the field of~~,

$$\vec{\tau} = \vec{\mu_m} \times \vec{H}$$

$$\Rightarrow \tau = \mu_m H \sin\theta$$

⇒ Work done in rotating the dipole through small angle $(-d\theta)$.

$$dw = \tau(-d\theta)$$

(ie) clockwise

$$= -\mu_m H \sin\theta d\theta$$

$$dw = H(-\mu_m \sin\theta d\theta) \text{ —— ②}$$

Using eqn. ①, we get

$$\Rightarrow dw = H dM \quad \text{(area of the strip)}$$

Total work done in taking the sub^n. through complete cycle of magnetisation and demagnetisation is,

$$w = \int dw = \int H dM$$

$$w = \text{area of } M-H \text{ loop} \text{ —— ③}$$

*Total energy dissipated during one cycle of magnetisation and demagnetisation is given by M-H loop.

⇒ Hysteresis loss = area of M-H loop.

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* Ferrimagnetism:— is a subⁿ characterized by magneti-
-zation in which two types of magnetic moment and of ions of unequal magnetic moment are
polarized in opposite direction,

↑↑↑ Ex:- Al, Co, Ni, Zn etc.
↑↑↑.

## ** Domain Theory of ferromagnetism:—

In ferromagnetic subⁿ the interaction betⁿ different diff. atoms in the neighbourhood is very complex is called Exchange interaction. Due to

Exchange interaction, the large no. of atomic dipoles get aligned in the same directⁿ. They are so strong even thermal vibration can't destroy their alignment.

The small region in which all the atoms have their magnetic moments aligned are called domains.

Each arrow shows the total magnetic moment of the domain. A domain has volᵐ of order of $10^{-2}cm^3$ to $10^{-6}cm^3$ may contain about $10^{16}$ or more atomic dipoles. The ferromagnetic substances are intrinsically magnetised even in the absence of magnetic field and net magnetic moment is zero.

When an External magnetic field is applied the atoms domain are aligned themselves along the directⁿ of magnetic field. It means the domains in which atoms ~~were not~~ are aligned in the direction of applied magnetic field expand at the cost of other domain in which atoms were not aligned.

for External field, the atoms are partially aligned
But the strength of field is increase.

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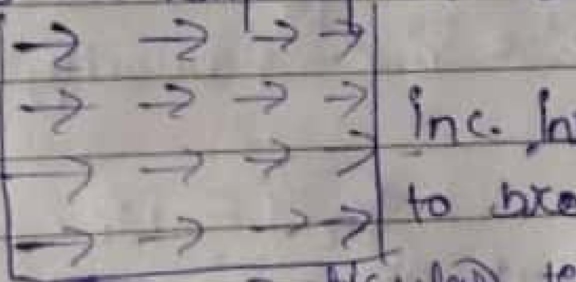
At very high field, all the domains get aligned further there is no increase in the magnetisation.

Decreasing Magnetic field: all the domains do not retain their original state.

As the temp. of the material gets raised, its internal energy increases and this causes all the atoms




to break lose from their neighbours and at a particular temp called Curie temp.

All the atomic dipoles loose their alignment.

(iii) Why is ferromagnetism not found in liquids and gases?

Ans:- Ferromagnetism occurs when many atoms can anisotropically into its domain and give a net magnetic dipole moment in solids. Then, the ferromagnetism are not found in liquid and gases due to thermal fluctuations.

(iv) What is ferrites. Which type of ferrites. Uses of ferrites?

Ans:- Ferrites:- The difference betn the magnetic moments of two adjascent atom is not equal to zero. These having properties are known as ferrites.

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• Ferrites are non-conducting subn. which are derived from an ion oxide.

Two types of ferrites:-

(i) Soft ferrites:- Low coercivity, high retentivity occurs.
Eg:- NiZn, MgZn, etc.

(ii) Hard ferrites:- High coercivity, high retentivity occurs. These are the parmanent magnets.
Eg:- Ba ferrites and strontium ferrites

Uses of ferrites:-

(i) Ferrites are poor conductor of Electricity. These are most suitable to make core of transformers. Soft ferrites are used to make purposes.

(ii) The hysteresis loop of Mg-Mn ferrites is nearly rectangular. Due to Mg-Mn ferrites are used in memory circuits of computers.

(iii) The Coercivity of hard ferrites is very high. Due to large value of Coercivity. These ferrites are used to make permanent magnets.

(v):- What is an irrotational fields? Give two Examples?

Ans:- Irrotational field :— A field whose curl is zero is called Irrotational field. $\vec{A}$ is an irrotational field. $\vec{\nabla}\times\vec{A}=0$.

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Eg:- Irrotational field is, Electrostatic field and Gravitational field.

(vi) What is the physical interpretation of gradient of a scalar function?

Ans:- The gradient of a scalar function $\phi(x,y,z)$ at a point represents a vector field whose magnitude is equal to the maximum spatial rate of change of field $\phi$ at that point and is directed along max^m rate of change.

$$\text{grad.}\,\phi = \left(\frac{d\phi}{dx}\right)_{max.}$$

(vii) Is Volume charge density invariant?

Ans:- The vol^m charge density of a body having charge $Q$ in frame $S$ is given by, PAGE-181

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# CHAPTER - 10

## Maxwell's Equations and Electromagnetic wave Equation :-

① **Displacement current :-** The changing the Electric field produces the Electric current. In betn. the plates of capacitor, changing the flux during the charging of the capacitors.

∴ The current is produced betn. the plates of capacitor. Maxwell called this current. It is called displacement current.

The SI unit of Displacement current is :- Ampere (A).

Acc. to Ampere's circuital law in the differential form:-

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} \quad \text{--- (1)}$$

Taking the divergence on b.s.

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0(\vec{\nabla}\cdot\vec{J}) \quad \text{--- (2)}$$

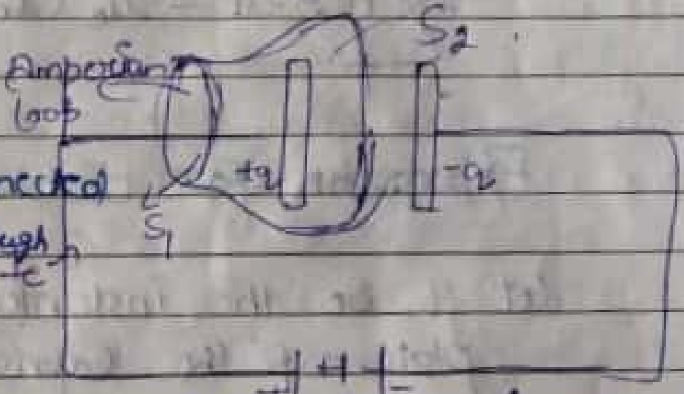
Divergence of curl is zero. then, $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B})=0$.

Now, Acc. to Eqn. of Continuity:— $\vec{\nabla}\cdot\vec{J} = -\frac{d\rho}{dt}$.

$\vec{\nabla}\cdot\vec{J}=0$ for steady currents.
182 and $\vec{\nabla}\cdot\vec{J}\neq 0$ for non-steady currents or Varying currents.

Ampere's circuital law in diff. Eqn. ① is Valid Steady current but not non-steady current. Then, Ampere's circuital law needs modi--fication for varying currents.

- Consider a parallel plate capacitor, connected to battery ~~with~~ through the wires.




⟹ When Conduction current $I_c$ passes in the wires Connecting in the parallel plate Capacitor. The charge on positive plate capacitor changes but no Conduction current exist betn. the gap ~~of~~ the parallel plate capacitor.

Consider, Amperian loop consisting two surfaces $S_1$ and $S_2$, around positively charged plate of the Capacitor.

For $S_1$ Surface:—

$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_c$ { $I_c$ be the Condn. Current Enclosed by the surface $S_1$ }.

for surface $S_2$,

$\oint \vec{B}\cdot d\vec{l} = 0$ { bec. no condn. current enclosed by the surface $S_2$ }.

Maxwell resolved this problem by postulating that changing electric field plate of capacitor is equivalent to current through the gap betn. the plate of the capacitor.

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Maxwell modified the Ampere's circuital law is called Maxwell - Ampere's law.

$$\Rightarrow \boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 (I_c + I_d)}$$

## Expression for displacement current :-

Let q be the instantaneous charge of the plate of the capacitor is,

$$q = CV \quad -① .$$

$$\boxed{C = \frac{q}{V}}$$

$$\Rightarrow C = \frac{\epsilon_0 A}{d} \quad -②$$

now, A is the area of each plate of the capacitor and d is the distn of the seperation betn. the plate of capacitor.

If, V be the instantaneous potential diff. betn. the plates of the capacitor, magnitude of the electric field in the gap betn. the plates.

$$\boxed{E = \frac{v}{d}} \Rightarrow \boxed{v = E\,d} \quad -③$$

Using Eqn. ② and ③ in Eqn ①,

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$$q = \left[\frac{\varepsilon_0 A}{d}\right] Ed = \varepsilon_0 A E \qquad \left\{\boxed{E = \frac{q}{\varepsilon_0}}\right.$$

$$E = \frac{q}{\varepsilon_0 A} \quad -④$$

Electric flux through the surface $S_2$ of Amperian loop is :-

$$\phi_E = EA = \left[\frac{q}{\varepsilon_0 A}\right] A = \frac{q}{\varepsilon_0} \quad ⑤$$

$$\Rightarrow \frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0}\frac{dq}{dt} = \frac{1}{\varepsilon_0}(I_d).$$

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

$$= \varepsilon_0 \frac{d}{dt}(EA) \Rightarrow \boxed{I_d = \varepsilon_0 A \frac{dE}{dt}} \quad ⑥$$

i.e. which is the Expression for the displacement current.

**Displacement current density :** $J_d = \frac{I_d}{A}$

$$\boxed{J_d = \frac{1}{A} \times \varepsilon_0 A \frac{dE}{dt}} \Rightarrow J_d = \varepsilon_0 \frac{dE}{dt} ⑦$$

Ear In Dielectric medium of permittivity $\epsilon$ fills the gap betn the plates of the capacitor. ⑥ and ⑦ eqn.

$$I_d = \epsilon A \frac{dE}{dt} \quad -⑧$$

$$\Rightarrow \boxed{J_d = \frac{I_d}{A} = \epsilon \frac{dE}{dt}} \quad -⑨.$$

Maxwell - Ampere circuital law can be written as :-

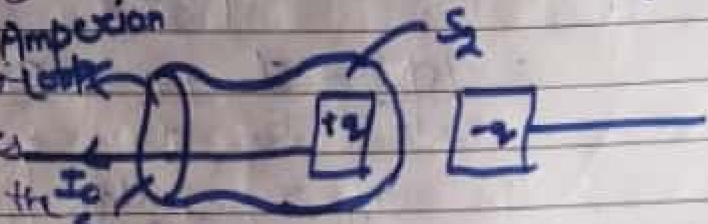
$$\oint \vec{B}\cdot\vec{dl} = \mu_0\left[I_c + \epsilon_0 A \frac{dE}{dt}\right].$$

$$= \mu_0\left[I_c + \epsilon_0 \frac{d\phi_e}{dt}\right].$$

## ② Continuity of Current :-

185 The sum of conduction current ($I_c$) and displacement current ($I_d$) is continous along the closed path, the individual conduction current and displacement current may not be continous along closed path.

Amperian loop consisting two surfaces $S_1$ and $S_2$ around the positive plate of the capacitor.




$$\text{Now, } \boxed{I_c + I_d = I_c + \epsilon_0 \frac{d\phi_E}{dt}} \quad -① .$$

Surface $S_1$ encloses the conduction current. But, charge is not accumulated on the wire. there is no change in the electric flux in the closed surface $S_1$ i.e $\frac{d\phi_E}{dt} = 0$.

$$\text{Now, } \boxed{I_c + I_d = I_c = \frac{dq}{dt}} \quad -② ,$$

There is no conduction current ~~betn~~ the gap betn the plates of capacitor ($I_c = 0$) but,

- Electric flux through the closed surface exist bec. of the accumulating of the charge on the positive plate of the capacitor.

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$$I_c + I_d = I_d = \varepsilon_0 \frac{d\phi_E}{dt} \quad \text{--- (3)}$$

$$\phi_E = E.A = \left[\frac{q}{\varepsilon_0 \cancel{A}}\right]\cancel{A} \Rightarrow \phi_E = \frac{q}{\varepsilon_0}$$

$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$ $\quad \Rightarrow I_d = \varepsilon_0 \frac{d\phi_E}{dt} \Rightarrow I_d = \cancel{\varepsilon_0}\frac{d}{dt}\left(\frac{q}{\cancel{\varepsilon_0}}\right)$

$I_d = \cancel{\varepsilon_0}\frac{d}{dt}\frac{q}{\cancel{\varepsilon_0}}$ $\Rightarrow I_d = \frac{dq}{dt}$

$$\boxed{I_d = \frac{dq}{dt}} \quad \text{--- (4)}$$

Eqn. (3) and (4) is clear that the sum of conduction current and displacement current is continous along any closed path. Conduction current and displacement current are Equal.

## (3) Differential form of Ampere - Maxwell law :—

Ampere's circuital law is :-

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I = \mu_0 \oiint_S \vec{J}\cdot d\vec{s} \quad \text{--- (1)}$$

Acc. to stoke's theorem :—

$$\oint_C \vec{B}\cdot d\vec{l} = \oiint_S (\vec{\nabla}\times\vec{B})\cdot d\vec{s} \quad \text{--- (2)}$$

$\Rightarrow$ using Eqn. (2) is (1) Eqn.,

$$\Rightarrow \oiint_S (\vec{\nabla}\times\vec{B})\cdot d\vec{s} = \mu_0 \oiint_S \vec{J}\cdot d\vec{s}$$

$$\Rightarrow \oiint_S \vec{\nabla}\times\vec{B} = \oiint_S \mu_0 \vec{J}\cdot d\vec{s}$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}}$$

$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$ — ③ This Eqn. is valid for steady currents. Let us Examine the non-steady current is :—

- Acc. to Gauss's law in ~~mag~~ Electrostatics :—

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$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \rho = \varepsilon_0(\vec{\nabla}\cdot\vec{E}) \quad \text{— ④}$$

Acc. to Eqn. of Continuity :—

$$\vec{\nabla}\cdot\vec{J} = \frac{-\partial\rho}{\partial t} \quad \text{— ⑤}$$

Eqn. ④ in ⑤,

$$\Rightarrow \vec{\nabla}\cdot\vec{J} = -\frac{\partial}{\partial t}(\varepsilon_0(\vec{\nabla}\cdot\vec{E}))$$

$$\boxed{\Rightarrow \vec{\nabla}\cdot\vec{J} = -\vec{\nabla}\cdot\varepsilon_0\frac{\partial \vec{E}}{\partial t}} \quad \text{— ⑤}$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\left[\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right] = 0} \quad \text{— ⑥}$$

which is the Eqn. of Continuity for non-steady currents.

Maxwell suggested that in Eqn. ③, $\vec{J}$ must be replaced by $\left(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$ for non-steady currents

For non-steady currents :— Eqn ③,

$$\Rightarrow \vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\boxed{\Rightarrow \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}} \quad \text{— ⑦}$$

$\Rightarrow$ Eqn. ⑦ is called diff. form of Ampere-Maxwell law

# Maxwell's Equation:—

188 Maxwell gaves the set of four fundamental Eqn. describing the Electric field and magnetic fields and their sources. This Eqn. is called maxwell's Eqn.

## (I) Maxwell's Eqn. in differential form:

**(a). Gauss's law for Electrostatics:—** This law relates the Electric field and charge produces on it.

$$\text{div}\,\vec{E} = \vec{\nabla}.\vec{E} = \rho/\epsilon_0$$ , This law signifies that Electric flux through the closed surface is Equal to the Enclosed by the closed surface.

Eqn. (1) is called maxwell's first Eqn.

**(b). Gauss's law for magnetostatics:—**

$$\text{div}\,\vec{B} = \vec{\nabla}.\vec{B} = 0.$$

This law signifies that magnetic monopole does not Exist.

**(c). Maxwell Ampere's circuital law:—**

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{d\vec{E}}{dt} = \mu_0\vec{J} + \mu_0\frac{d\vec{D}}{dt}.$$

This law signifies that the changing the electric field produces the magnetic field and this changing magnetic field produces the Electric field.

(4). Faraday's law of Electromagnetic Induction:—

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}} \quad \text{--- (4)}$$

This law signifies that the set up due to magnetic field. Eqn. (4) is called maxwell's Eqn. for 4th Eqn.

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In free space, $\vec{J}=0$ and $\rho=0$:

Maxwell's Eqn. are:—

$$\boxed{\vec{\nabla}\cdot\vec{E}=0}$$

$$\boxed{\vec{\nabla}\cdot\vec{B}=0}$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{d\vec{E}}{dt}}$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}}$$

## Maxwell's Equations in Integral form.

(1). Gauss's law for Electrostatics:—

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon$$

$$\Rightarrow \vec{\nabla}\cdot(\varepsilon_0\vec{E}) = \rho.$$

$$\Rightarrow \varepsilon_0\vec{E} = \vec{D}. \Rightarrow \vec{\nabla}\cdot\vec{D} = \rho.$$

Integrating over the volm. of a surface:

$$\iiint_V (\vec{\nabla}\cdot\vec{D})dv = \iiint_V \rho\, dv$$

$$\Rightarrow \iiint_V (\vec{\nabla}\cdot\vec{D})dv = \oint\!\!\oint_S (\vec{D}\cdot d\vec{s}). \quad \text{(Gauss's div. theorem)}$$

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$\Rightarrow \iiint_V \rho\, dv = Q_{enc}$ (Total charge enclosed in the vol$^m$).

$\Rightarrow \boxed{\oiint_S (\vec{D}\cdot d\vec{s}) = Q_{enc}}$ which is called integral form of Gauss's law of Electrostatics (Kalpana Kumari)

② . Gauss's law for magnetostatics :—

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0} \quad \text{--- (1)}$$

Integrating over a closed vol$^m$ of a surface,

$$\iiint_V (\vec{\nabla}\cdot\vec{B})\, dv = 0 \quad \text{--- (2)}$$

Acc. to Gauss's div theorem :—

$$\iiint_V (\vec{\nabla}\cdot\vec{B})\, d\vec{s} = \oiint_S (\vec{B}\cdot d\vec{s}) \quad \text{--- (3)}$$

Put ③ in ② Eqn.

Now, $\boxed{\oiint_S \vec{B}\cdot d\vec{s} = 0}$, which is the integral form of Gauss's law of magnetostatics.

③ . Maxwell - Ampere's circuital law :—

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}.$$

Integ. over a (closed vol$^m$) surface enclosed by closed surface :—

$$\oiint_S (\vec{\nabla}\times\vec{H})\cdot d\vec{s} = \oiint_S \vec{J}\cdot d\vec{s} + \oiint_S \frac{\partial \vec{D}}{\partial t}\cdot d\vec{s}.$$

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Acc. to stoke's theorem:—

$$\oiint_S (\vec{\nabla}\times\vec{H})\cdot d\vec{s} = \oint_C \vec{H}\cdot d\vec{l}$$

$$\boxed{\oiint_S \vec{J}\cdot d\vec{s} = I_{enc.}}$$

Now, $$\boxed{\oint_C \vec{H}\cdot d\vec{l} = I_{enc.} + \frac{d}{dt}\oiint_S \vec{D}\cdot d\vec{s}} \quad (3)$$

• which is the integral form of Maxwell-Ampere's circuital law.

(4). Faraday's law of Electromagnetic Induction:—

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Intg. over a surface enclosed by closed loop

$$\oiint_S (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -\oiint_S \frac{d}{dt}\vec{B}\cdot d\vec{s} = -\oiint_S \frac{d}{dt}(\vec{B}\cdot d\vec{s})$$

⟹ Acc. to stoke's theorem:—

$$\oiint_S (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = \oint_C \vec{E}\cdot d\vec{l}$$

$$\Rightarrow \boxed{\oint_C \vec{E}\cdot d\vec{l} = -\oiint_S \frac{d}{dt}(\vec{B}\cdot d\vec{s})} \quad (4)$$

• which is called Integral form of faraday's law of ~~the~~ Electromagnetic Induction.

Q1:- What do you understand by Electromagnetic wave? Give their imp properties?

Ans:- Electromagnetic waves:- Electromagnetic waves consists of time varying Electric and magnetic fields which are mutually ⊥ar to each other and also perpendicular to the direct$^n$ of wave propagation of wave.

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⇒ Properties of Electromagnetic waves:-

(i) Electromagnetic waves do no require any material medium for their propagation.

(ii) Electromagnetic waves are transverse in nature.

(iii) Electromagnetic waves travel in free space with the velocity $3 \times 10^8 ms^{-1}$.

(iv) Electromagnetic waves obey principle of super-position.

Q2:- Define Poynting vector? What does it represents? Give its SI unit?

Ans:- Poynting Vector:- The cross product of Electric field $\vec{E}$ and magnetic field vector $\vec{H}$ is called poynting vector.

It is denoted by $\vec{S}$.

$$\boxed{\vec{S} = \vec{E} \times \vec{H}}$$

Poynting vector gives the time rate of flow of Electromagnetic waves energy per unit area of the medium.

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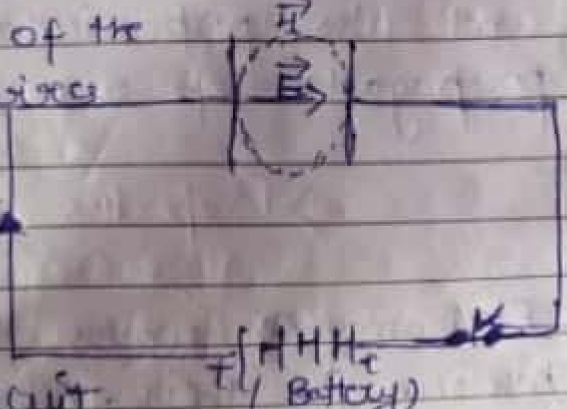
# Poynting Vector:—

Consider a circuit to flow of the Conduction current in the wires of the circuit. Circuit Containing parallel plate Capacitor Connecting to a battery. Consider a key K is also present in this circuit.




When the key K is closed, the capacitor gets charged.

⇒ A Conduction current I flows through the circuit and displacement current of density $\frac{\partial \vec{D}}{\partial t}$ is set up in betn. the plates of the Capacitor. The poynting vector is everywhere directed into the Electric field as magnetic lines of force are circular and Electric field is directed downward inside the plates.

⇒ SI unit of poynting vector is $Wm^{-2}$.

## Poynting theorem:— 

This theorem states that the rate of work done on charges in Vol$^m$ $v$ enclosed by surface $S$ by the Lorentz force is equal to rate of dec. of energy stored in Electromagnetic field minus the rate at which energy flows out of the surface $S$.

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$$\frac{dw}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)dv - \frac{1}{\mu_0}\oint_S (\vec{E}\times\vec{B})\cdot d\vec{s}$$

### Proof :—

(fractional charges)

Let us consider some charges and current distribution in Vol$^m$ $v$ enclosed by surface $S$. Let $\vec{E}$ and $\vec{B}$ are the electric field and magnetic field as produced in the region of surface. Charges moves slightly through a dist$^n$ $dl$ small dist$^n$ $dt$.

Lorentz force acting on a charge $dq$.

$$\vec{F} = dq\,[\vec{E} + (\vec{v}\times\vec{B})] \Rightarrow \underset{(\text{small charges})}{dq}\,[\vec{E} + (\vec{v}\times\vec{B})]$$

⇒ Work done by the Lorentz force in displacing charge $dq$ through $d\vec{l}$.

$$dw = \vec{F}\cdot d\vec{l} = dq\,[\vec{E} + (\vec{v}\times\vec{B})]\cdot d\vec{l} \quad \text{——} (1)$$

If $\vec{E}$ ~~is the~~ acts for small time $dt$ on the charge, then velocity of charge is,

$$\vec{v} = \frac{d\vec{l}}{dt} \quad \text{or} \quad d\vec{l} = \vec{v}\,dt$$

Substitute in Eqn. (1) we get

$dw = dq\,[\vec{E} + (\vec{v} \times \vec{B})]\cdot\vec{v}\,dt$

$dw = [\vec{E}\,dq + (\vec{v} \times \vec{B})\,dq]\cdot\vec{v}\,dt$

$\Rightarrow dw = (\vec{E}\cdot\vec{v})\,dq\,dt + \underbrace{(\vec{v} \times \vec{B})\cdot\vec{v}\,dq\,dt}$

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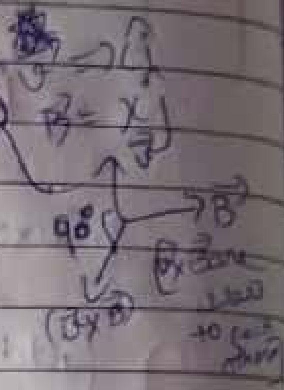
• $(\vec{v} \times \vec{B})\cdot\vec{v} = 0$  { $\vec{v}$ and $(\vec{v} \times \vec{B})$ are ⊥ to each other

$\boxed{dw = (\vec{E}\cdot\vec{v})\,dq\,dt}$



If $\rho$ = vol$^m$ charge density,

$\rho = \frac{dq}{dv} \Rightarrow dq = \rho\,dv$.

Now, $dw = (\vec{E}\cdot\vec{v})\,\rho\,dv\,dt$

$\Rightarrow dw = (\vec{E}\cdot\vec{v}\rho)\,dv\,dt$.

$(\vec{v}\rho = \vec{J})$  $\Rightarrow \boxed{\vec{v}\rho = \vec{J}}$

$\vec{v}_f = \frac{x}{t}\cdot\frac{q}{v} = \frac{xq}{tAx} = \frac{I}{A} = \vec{J}$.

$dw = (\vec{E}\cdot\vec{J})\,dv\,dt$.

$\boxed{\frac{dw}{dt} = (\vec{E}\cdot\vec{J})\,dv}$

$\Rightarrow$ Rate at which total work is done on all the charges in vol$^m$ $v$.

$$\frac{dw}{dt} = \iiint_V (\vec{E}\cdot\vec{J})\,dv \quad \text{---} \ (2)$$

$\Rightarrow \vec{E}\cdot\vec{J}$ represents work done per unit time per unit vol$^m$ by $\vec{E}$ or power dissipated per unit vol$^m$.

Work done $\vec{B}$ by zero,

$$\vec{\nabla}\times\vec{B} = \mu_0\left[\vec{J} + \epsilon_0\frac{d\vec{E}}{dt}\right]$$

$$\frac{1}{\mu_0}(\vec{\nabla}\times\vec{B}) = \vec{J} + \epsilon_0\frac{d\vec{E}}{dt}$$

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$$\vec{J} = \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B}) - \epsilon_0\frac{d\vec{E}}{dt}.$$

$$\Rightarrow \vec{E}.\vec{J} = \frac{1}{\mu_0}\vec{E}.(\vec{\nabla}\times\vec{B}) - \epsilon_0\vec{E}.\frac{d\vec{E}}{dt} \quad (3).$$

Using vector identity :—

$$\Rightarrow \vec{\nabla}.(\vec{A}\times\vec{B}) = \vec{B}.(\vec{\nabla}\times\vec{A}) - \vec{A}.(\vec{\nabla}\times\vec{B}).$$

$$\vec{A}.(\vec{\nabla}\times\vec{B}) = \vec{B}.(\vec{\nabla}\times\vec{A}) - \vec{\nabla}(\vec{A}\times\vec{B}) \quad (4).$$

Using Eqn. (4) in Eqn. (3),

$$\Rightarrow \vec{E}.\vec{J} = \frac{1}{\mu_0}\left[\vec{B}(\vec{\nabla}\times\vec{E}) - \vec{\nabla}.(\vec{E}\times\vec{B})\right] - \epsilon_0\vec{E}\frac{d\vec{E}}{dt}$$

Acc. to Faraday's law :—

$$\Rightarrow \vec{\nabla}\times\vec{E} = \frac{-d\vec{B}}{dt}$$

$$\Rightarrow \vec{E}.\vec{J} = \frac{1}{\mu_0}\left[\vec{B}.\left(\frac{-d\vec{B}}{dt}\right) - \vec{\nabla}.(\vec{E}\times\vec{B})\right] - \epsilon_0\vec{E}.\frac{d\vec{E}}{dt}$$

$$\Rightarrow \vec{E}.\vec{J} = -\frac{1}{\mu_0}\left[\vec{B}.\frac{d\vec{B}}{dt} - \epsilon_0\vec{E}.\frac{d\vec{E}}{dt} - \frac{1}{\mu_0}\vec{\nabla}.(\vec{E}\times\vec{B})\right].$$

$$= -\frac{1}{2\mu_0} 2\vec{B}\cdot\frac{\partial \vec{B}}{\partial t} - \frac{1}{2}\varepsilon_0 2\vec{E}\cdot\frac{\partial \vec{E}}{\partial t} - \frac{1}{\mu_0}\vec{\nabla}\cdot(\vec{E}\times\vec{B}) - (1)$$

$$\frac{\partial}{\partial t}(\vec{E}\cdot\vec{E}) = \vec{E}\cdot\frac{\partial \vec{E}}{\partial t} + \vec{E}\cdot\frac{\partial \vec{E}}{\partial t} = 2\vec{E}\cdot\frac{\partial \vec{E}}{\partial t}.$$

$$\frac{\partial}{\partial t}(\vec{B}\cdot\vec{B}) = 2\vec{B}\cdot\frac{\partial \vec{B}}{\partial t}.$$

$$\vec{E}\cdot\vec{J} = -\frac{1}{2\mu_0}\frac{\partial}{\partial t}(\vec{B}\cdot\vec{B}) - \frac{1}{2}\varepsilon_0\frac{\partial}{\partial t}(\vec{E}\cdot\vec{E}) - \frac{1}{\mu_0}\vec{\nabla}\cdot(\vec{E}\times\vec{B}).$$

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$$\vec{E}\cdot\vec{J} = -\frac{1}{2}\left[\varepsilon_0\frac{\partial E^2}{\partial t} + \frac{1}{\mu_0}\frac{\partial B^2}{\partial t}\right] - \frac{1}{\mu_0}\vec{\nabla}(\vec{E}\times\vec{B}).$$

Substituting $\vec{E}\cdot\vec{J}$ in eqn. (2)

$$\frac{dw}{dt} = \iiint_V -\frac{1}{2}\left[\varepsilon_0\frac{\partial E^2}{\partial t} + \frac{1}{\mu_0}\frac{\partial B^2}{\partial t}\right] - \frac{1}{\mu_0}\vec{\nabla}\cdot(\vec{E}\times\vec{B})\,dv.$$

$$\Rightarrow \frac{dw}{dt} = -\frac{\partial}{\partial t}\iiint_V \frac{1}{2}\left[\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right]dv - \frac{1}{\mu_0}\iiint_V \vec{\nabla}\cdot(\vec{E}\times\vec{B})\,dv$$

$\Rightarrow$ Acc to divergence theorem:-

$$\iiint_V \vec{\nabla}\cdot(\vec{E}\times\vec{B})\,dv = \oiint_S (\vec{E}\times\vec{B})\,d\vec{S}$$

$$\frac{dw}{dt} = -\frac{\partial}{\partial t}\iiint_V \frac{1}{2}\left[\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right]dv - \frac{1}{\mu_0}\oiint_S (\vec{E}\times\vec{B})\cdot d\vec{S}. - (7).$$

$\Rightarrow \frac{dw}{dt}$ represents the rate at which work is done on all the charges in vol$^m$. $V$.

This is the required Expression for Poynting theorem.

$$\iiint \frac{1}{2}\left[\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right] dv = U_{em}$$ represents the total Energy stored in the Electric and magnetic field.

198 $\frac{1}{\mu_0} \oiint_S (\vec{E} \times \vec{B}) . d\vec{s}$ represents the rate at which Energy flows out of the surface S Enclosing the vol. v

* ⇒ The vector $\frac{1}{\mu_0}(\vec{E} \times \vec{B})$ represents the amount of energy flow per unit time per unit area by the field and is called Poynting Vector.

$$\boxed{\vec{S} = \frac{1}{\mu_0}\left[\vec{E} \times \vec{B}\right].}$$

## Electromagnetic wave Equations in a medium having finite permittivity and permeability but with Conductivity $\sigma = 0$

• Medium is a subⁿ that makes possible transfer of energy from one location to another, especially in waves called medium.

**Wave Eqn. Satisfied By $\vec{E}$ and $\vec{B}$ in dielectric medium:—**

Dielectric medium:— It may be defined as that a medium in which the electric conductivity is not equal to zero yet it is not good conductor.

Dielectric:— Insulating material, a very poor conductor of electric current.

Ex:— plastics, Glass, various metal oxides and Even dry air is also Ex. of dielectric.

In a dielectric medium, there are no free charges:—

$\rho = 0$ (volᵐ charge density).

$\sigma = 0$ (Conductivity).

$\vec{J} = \frac{I}{A}$

$\rho = \frac{q}{\frac{t}{a}}$ $\Rightarrow \vec{J} = \frac{q}{t} \times \frac{1}{a}$

$I = \frac{q}{t}$

$$\vec{J} = \sigma \vec{E} = 0$$

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

$\vec{B} = \mu_0 \vec{H}$ → (magnetic field intensity)

magnetic Induction

(magnetic permeability)

vacuum permittivity of free space → permeability

Where $\epsilon$ and $\mu$ have finite values. Maxwell's Parameter Equation's are:—

$\epsilon_0 = 8.85 \times 10^{-12} Fm^{-1}$

$$\Rightarrow \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} = 0 \quad \text{——— (1)}$$

$$\Rightarrow \vec{\nabla} \cdot \vec{B} = 0 \quad \text{——— (2)}$$

(No zero)

$$\Rightarrow \vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} \quad \text{——— (3)}$$

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$$\Rightarrow \vec{\nabla}\times\vec{B} = \mu\left[\vec{J} + \epsilon\frac{d\vec{E}}{dt}\right]$$

$$= \mu\epsilon\frac{d\vec{E}}{dt} \quad (4) \qquad \left\{\begin{array}{l}\epsilon\vec{E} = \vec{D} \\ \vec{J} = 0\end{array}\right.$$

(a) Wave Equation for Electric field vector $\vec{E}$ :-

Taking curl of both sides of Eqn. (3) we get.

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$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\vec{\nabla}\times\frac{d\vec{B}}{dt}.$$

Using Identity:-

$$\Rightarrow \vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B}).$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \vec{E}(\vec{\nabla}\cdot\vec{\nabla}) = -\vec{\nabla}\times\frac{d\vec{B}}{dt}.$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \vec{E}\nabla^2 = -\vec{\nabla}\times\frac{d\vec{B}}{dt}$$

$\nabla^2$

$\vec{\nabla}\cdot\vec{\nabla} = \nabla^2$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{d}{dt}(\vec{\nabla}\times\vec{B})$$

Using Eqn. (1) and Eqn. (4).

$$0 - \nabla^2\vec{E} = -\frac{d}{dt}\left[\mu\epsilon\frac{d\vec{E}}{dt}\right].$$

$$\cancel{-}\nabla^2\vec{E} = \cancel{-}\mu\epsilon\frac{d^2\vec{E}}{dt}.$$

$$\Rightarrow \boxed{\nabla^2\vec{E} = \mu\epsilon\frac{d^2\vec{E}}{dt}}$$

This is general Electromagnetic wave Eqn. in terms of electric field vector $\vec{E}$ in a dielectric medium (non-Conducting).

(b) Wave Eqn. for magnetic field ($\vec{B}$).

Taking curl of both sides of Eqn. (4),

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\times\mu\epsilon\frac{\partial\vec{E}}{\partial t}.$$

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$$\Rightarrow \vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C})-\vec{C}(\vec{A}\cdot\vec{B}).$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{\nabla}) = \vec{\nabla}\times\mu\epsilon\frac{\partial\vec{E}}{\partial t}.$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = \mu\epsilon\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}).$$

Using Eqn. (2) and (3) we get

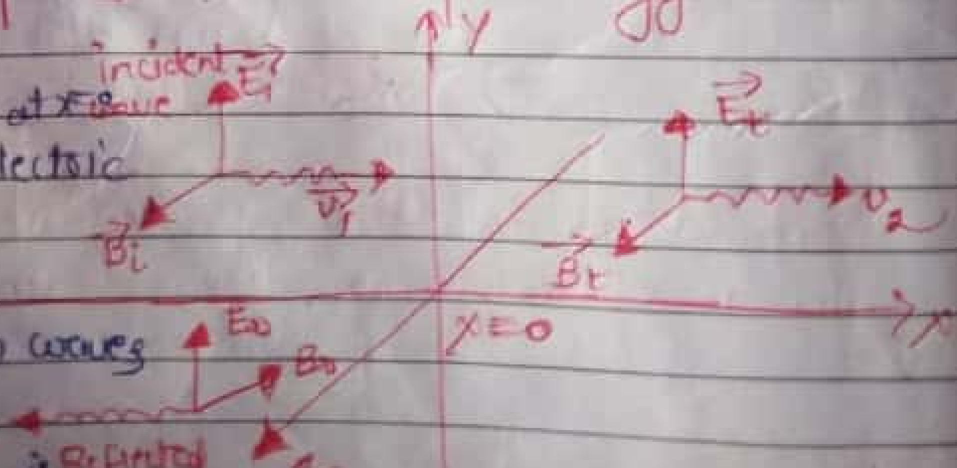
14 $$0-\nabla^2\vec{B} = \mu\epsilon\frac{\partial}{\partial t}\left[\frac{-\partial\vec{B}}{\partial t}\right].$$

$$\boxed{\nabla^2\vec{B} = \mu\epsilon\frac{\partial^2\vec{B}}{\partial t^2}}.$$

This is the general electromagnetic wave Eqn. in terms of magnetic field $\vec{B}$ in dielectric medium.

## * Reflection and Transmission of E.M wave at Normal incidence at a plane Boundary of two Dielectrics or law of Conservation of Energy.

Consider an interface at x=0 wave separating two dielectric media 1 and 2.




Let a plane e.m waves travelling along the axis with $\vec{E}$ along y-axis and $\vec{B}$ vector along z-axis strikes the plane interface at

Physics:

[Solution of wave Equation in a Conducting medium:]

Date:
Page No. $-p^2+2i\alpha\beta = \mu\varepsilon\omega^2 i\mu\sigma\omega$

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* Wave Eqn. for electric field vector $\vec{E}$ in a Conducting medium is:—

$$\nabla^2\vec{E} - \mu\sigma\frac{\partial \vec{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0 \quad \text{--- (1)}$$

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Let the wave is propagating along x-direct$^n$. with $\vec{E}$ along y-axis and $\vec{B}$ vector along z-axis.

i.e $E_y \neq 0, E_x = 0$ and $E_z = 0$

$\vec{E}$ is the funct. of $x$ and $t$ only. There is no vib. of $\vec{E}$ y and z-axis.

Eqn. (1) Can be written as:—

$$\frac{\partial^2\vec{E}}{\partial x^2} - \mu\sigma\frac{\partial \vec{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0 \quad \text{--- (2)}$$

Plane wave Sol$^n$. of this Eqn. is: $\vec{E} = E_0 e^{i(kx-\omega t)}\hat{j}$ — (3).

* Subst. this Eqn. in (2) we get.

$$\frac{\partial^2(E_0 e^{i(kx-\omega t)}\hat{j})}{\partial x^2} - \mu\sigma\frac{\partial}{\partial t}E_0 e^{i(kx-\omega t)}\hat{j} - \mu\varepsilon\frac{\partial^2}{\partial t^2}E_0 e^{i(kx-\omega t)}\hat{j} = 0.$$

$$i^2k^2E_0 e^{i(kx-\omega t)}\hat{j} - \mu\sigma(-i\omega)E_0 e^{i(kx-\omega t)}\hat{j} - \mu\varepsilon(i^2\omega^2)E_0 e^{i(kx-\omega t)} =$$

$$E_0 e^{i(kx-\omega t)}\hat{j}\left[-k^2 + i\mu\sigma\omega + \mu\varepsilon\omega^2\right] = 0.$$

$$-k^2 + i\mu\sigma\omega + \mu\varepsilon\omega^2 = 0$$

$$k^2 = \mu\varepsilon\omega^2 + i\mu\sigma\omega. \quad \text{--- (4)}.$$

This Eqn shows that the conducting medium, propagation constant $k$ is a Complex quantity

$$\text{Let } k = \alpha + i\beta. \quad \text{--- (5)}.$$

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where the real part $\alpha$ is called attenuation constt- and imaginary part $\beta$ is called phase constt.

Squaring Eqn. (5).

$$K^2 = \alpha^2 - \beta^2 + 2i\alpha\beta \quad — (6)$$

Comparing real and imaginary part of Eqn. (4) and (6) in Eqn.

$$\alpha^2 - \beta^2 = \mu\epsilon\omega^2 \; — (7) \qquad 2\alpha\beta = \mu\sigma\omega \; — (8) \qquad \beta = \frac{\mu\sigma\omega}{2\alpha} \; (9)$$

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Substituting Eqn. (9) in Eqn (7) we get

$$\alpha^2 - \left(\frac{\mu\sigma\omega}{2\alpha}\right)^2 = \mu\epsilon\omega^2 \Rightarrow \alpha^2 - \frac{\mu^2\sigma^2\omega^2}{4\alpha^2} = \mu\epsilon\omega^2$$

Multi. or divide by $\alpha^2$:-

$$\Rightarrow \alpha^4 - \frac{\mu^2\sigma^2\omega^2}{4} = (\mu\epsilon\omega^2)\alpha^2 \Rightarrow \underset{a}{\alpha^4} - \underset{b}{(\mu\epsilon\omega^2)}\alpha^2 - \underset{c}{\frac{\mu^2\sigma^2\omega^2}{4}} = 0.$$

$$\Rightarrow \alpha^2 = \frac{\mu\epsilon\omega^2 \pm \sqrt{(\mu\epsilon\omega^2)^2 + 4\left(\frac{\mu^2\sigma^2\omega^2}{4}\right)}}{2} \Rightarrow \alpha^2 = \frac{\mu\epsilon\omega^2 \pm \sqrt{(\mu\epsilon\omega)^2 + \mu^2\sigma^2\omega^2}}{2}$$

$$\Rightarrow \alpha^2 = \frac{\mu\epsilon\omega^2 \pm \mu\epsilon\omega^2\sqrt{1 + \frac{\sigma^2}{\epsilon^2\omega^2}}}{2} \Rightarrow \text{Choosing the +ve sign}$$

$$\Rightarrow \alpha^2 = \frac{\mu\epsilon\omega^2 + \mu\epsilon\omega^2\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2}}{2} \Rightarrow \alpha^2 = \frac{\mu\epsilon\omega^2}{2}\left[1 + \sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2}\right]$$

$$\Rightarrow \alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[1 + \sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2}\right]^{1/2}, \text{ for a good conduc., } \frac{\sigma}{\epsilon\omega} >> 1,$$

neglecting 1 as compared to $\left(\frac{\sigma}{\epsilon\omega}\right)^2$, $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\frac{\sigma}{\epsilon\omega}\right)}$

$$\Rightarrow \alpha = \sqrt{\frac{\mu\sigma\omega}{2}} \Rightarrow \text{Subs. } \alpha \text{ in Eqn. (9)} \Rightarrow \beta = \frac{\mu\sigma\beta}{2} \times \sqrt{\frac{2}{\mu\sigma\omega}}$$

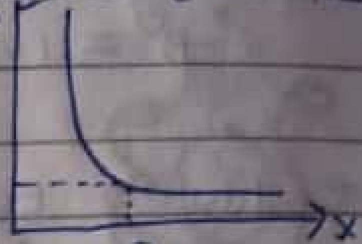
$$= \sqrt{\frac{\mu\sigma\omega}{2}} \Rightarrow \alpha = \beta = \sqrt{\frac{\mu\sigma\omega}{2}}$$ . Subst. the value of K from Eqn (5) in Eqn. (3).

$$\vec{E} = E_0 e^{i(Kx - \omega t)}\hat{j}$$

$$= E_0 e^{i(\alpha + i\beta - \omega t)}\hat{j} = E_0 e^{-\beta x} \times e^{i(\alpha x - \omega t)} \quad (10)$$

Eqn ⑩ is the Eqn. of plane e.m wave whose amplitude of $E_0 e^{-\beta x}$. This shows that the amplitude of e.m wave dec. exp. as wave travels in the ~~conducting medium~~.

* [Skin Depth: It states that the distance travelled by Electromagnetic wave or Electric $\vec{E}$ Field vector in a Conductor during which the amplitude of wave decays to $1/e$ times its value on the surface of the Cond.$^n$




It is denoted by $\delta$.

$$E_a(\delta) = \frac{1}{e} E_0 \quad \text{—①}$$

when electromagnetic wave travels through the conducting medium, the wave is attenuated. This mean that amplitude of the wave of both electric and magnetic field of e.m. waves decreases with dist$^n$.

Let wave is propagated along x-direct$^n$. Then, the amplitude of Electric Field in the Conductor is:—

$E_a(x) = E_0 e^{-\beta x}$, $E_0$ is the Electric Field at the surf. of the conductor., $\beta = \sqrt{\frac{\mu\sigma\omega}{2}}$, $\omega$ = angular freq. of wave, $\mu$ = permeability of Cond. and $\sigma$ = Conductivity of cond.

*Amplitude of Electric field at a dist$^n$ $x = \delta$.

$E_a(\delta) = E_0 e^{-\beta x}$ —③. From Eqn.(1) and (3) we get

$$E_0 e^{-\beta\delta} = \frac{E_0}{e} \Rightarrow E_0 e^{-\beta\delta} = e^{-1} E_0 \Rightarrow \beta\delta = 1.$$

$$\delta = \frac{1}{\beta} = \sqrt{\frac{2}{\mu\sigma\omega}} \quad \text{—④}$$

$\int \beta = \frac{1}{\delta}$ This Eqn shows that the skin depth is inversely proportional to the square root of

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the angular frequency. E.m wave of higher frequency travels small distn. through a conducting medium.



For perfect conductor $\sigma = \infty$, skin depth $\delta = 0$ ie It means that no e.m wave can penetrate through a perfect conductor.

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**Wave or Phase velocity of E.m waves in a conducting Medium or Conductor:—

The wave velocity of e.m wave in a medium is given by:—

$v = \frac{\omega}{K}$, $\omega$ is the angular frequency and K is propagation constt., For conducting medium, K is complex quantity.

$K = \alpha + i\beta$. —②, For a good conductor,

$\alpha = \left[\frac{\mu\sigma\omega}{2}\right]^{1/2}$ —③ As the wave velocity is always real so take part of K i.e i.e $K = \alpha$, Eqn ① becomes

$\boxed{v = \frac{\omega}{\alpha}}$ —①. Using Eqn. ③ we get.

$v = \frac{\omega}{\sqrt{\frac{\mu\sigma\omega}{2}}} = \left(\frac{2\omega^2}{\sigma\mu\omega}\right)^{1/2} = \sqrt{\frac{4\pi v}{\sigma\mu}}$ —④, Hence

$\omega = \frac{2\pi}{T}$

The wave or phase of velocity of Electromagnetic wave in a conducting medium or Conductor is depends upon its frequency. Ans.

# *Characteristic Impedence or Intrinsic Impedance or wave impedence of free space.

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**Definition:-** The ratio of magnitude of Electric field intensity ($\vec{E}$) to the magnitude of magnetic field intensity ($\vec{H}$).

$$Z_0 = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

S.I unit of characteristic impedence is ohm ($\Omega$)

**Derivation:-** The wave Equations for Electromagnetic waves in free space are

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t} \quad \text{--- (1)}$$

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t} \quad \text{--- (2)}$$

The plane wave solution of the above Eqn. can be written as:-

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)} \quad \text{--- (3)}$$

$$\vec{B} = \vec{B}_0 e^{i(\vec{k}\cdot\vec{x} - \omega t)} \quad \text{--- (4)}$$

For free space, Maxwell's third Eqn. is:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (5)}$$

Substituting Eqn. ③ and ④ in Eqn. ⑤

$$\vec{\nabla}\times\vec{E}_0 e^{i(\vec{K}\cdot\vec{x}-\omega t)} = -\frac{\partial}{\partial t}\vec{B}_0 e^{i(\vec{K}\cdot\vec{x}-\omega t)}$$

$$i\vec{K}\times\vec{E}_0 e^{i(\vec{K}\cdot\vec{x}-\omega t)} = -(-i\omega)\vec{B}_0 e^{i(\vec{K}\cdot\vec{x}-\omega t)}$$

$$\cancel{i}\vec{K}\times\vec{E} = \cancel{i}\omega\vec{B}$$

$$\Rightarrow \vec{K}\times\vec{E} = \omega\vec{B} \quad \text{—— ⑥}$$

$\vec{B}$ is ⊥ar to both $\vec{K}$ and $\vec{E}$. The vectors $\vec{E}, \vec{B}, \vec{K}$ form a right handed orthogonal set.

But $\vec{K} = K\hat{n}$, where $\hat{n}$ is a unit vector along the directⁿ of propagation of Em wave.

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Eqn. ⑥ can be written as :

$$K(\hat{n}\times\vec{E}) = \omega\vec{B} \Rightarrow \vec{B} = \mu_0\vec{H}$$

$$K(\hat{n}\times\vec{E}) = \mu_0\omega\vec{H} \Rightarrow \vec{H} = \frac{K}{\mu_0\omega}(\hat{n}\times\vec{E}),$$

$$\text{Now } K = \frac{\omega}{c}, \quad \vec{H} = \frac{\cancel{\omega}}{c\times\cancel{\omega}\mu_0}(\hat{n}\times\vec{E}).$$

$$\Rightarrow \vec{H} = \frac{1}{\mu_0 c}(\hat{n}\times\vec{E}). \Rightarrow \vec{H} = \frac{1}{\mu_0 c}|\vec{E}|.$$

In free space

$$\frac{|\vec{E}|}{|\vec{H}|} = \mu_0 c. \Rightarrow c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}.$$

$$\frac{\vec{E}}{|\vec{H}|} = \mu_0\frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

The unit of E is $\frac{\text{Volt}}{\text{metre}}$ and unit of H is $Am^{-1}$

The unit ratio of $\frac{|E|}{|H|}$ is Equal to $\frac{\text{Volt}}{\text{ampere}}$ = ohm.

classmate

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$\frac{|\vec{E}|}{|\vec{H}|}$ has same unit as those of impe-dence and is called as **characteristic or intrin-sic impedence of free space.**

It is denoted by $Z_0$.

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$$Z_0 = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} = 376.6 \text{ ohms}$$

$\approx$ 377 ohm.

Important Note:—

- Free space offers an impedence of 377 ohm to the e.m. wave travelling through it.

Also the ratio of $\frac{|\vec{E}|}{|\vec{H}|}$ is real and positive. $\vec{E}$ and $\vec{H}$ are the same phase of vibration. When the amplitude of $\vec{E}$ is max$^m$ then the amplitude of $\vec{H}$ is also max$^m$.

## * Characteristic impedence or Intrinsic impedence of a Dielectric medium to E.M wave.

**Definition:**— The ratio of magnitude of Electric field intensity to the amplitude of magnetic field intensity ($\vec{H}$).

$$Z_d = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\mu/\varepsilon}$$

**Derivation:—** The wave Eqn. for e.m waves in a dielectric medium is given by:



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$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{—} \; ①$$

$$\nabla^2 \vec{B} = \mu \epsilon \frac{\partial^2 \vec{B}}{\partial t^2} \quad \text{—} \; ②$$

The plane wave solutions of above Eqn. can be written as:—

$$\Rightarrow \vec{E} = \vec{E}_0 e^{i(\vec{K}\cdot\vec{x} - \omega t)} \quad ③$$

$$\Rightarrow \vec{B} = \vec{B}_0 e^{i(\vec{K}\cdot\vec{x} - \omega t)} \quad ④$$

For dielectric medium, Maxwell's third Eqn. is

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} \quad \text{—} \; ⑤$$

Substituting Eqns ③ and ④ in Eqn. ⑤ we get.

$$\vec{\nabla} \times \vec{E}_0 e^{i(\vec{K}\cdot\vec{x} - \omega t)} = -\frac{d}{dt} \vec{B}_0 e^{i(\vec{K}\cdot\vec{x} - \omega t)}$$

$$i\vec{K} \times \vec{E}_0 e^{i(\vec{K}\cdot\vec{x} - \omega t)} = -(-i\omega) \vec{B}_0 e^{i(\vec{K}\cdot\vec{x} - \omega t)}$$

$$i\vec{K} \times \vec{E} = i\omega \vec{B}.$$

$$\Rightarrow \vec{K} \times \vec{E} = \omega \vec{B}.$$

$\vec{B}$ is ⊥ar to both $\vec{K}$ and $\vec{E}$.

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and ... $E_1, E_0$ and ...

But $\vec{K} = K\hat{n}$, $\hat{n}$ is the unit vector along the direcⁿ of propagation of the Em waves

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$$K(\hat{n} \times \vec{E}) = \omega \vec{B}.$$

$$\vec{B} = \mu \vec{H} \Rightarrow K(\hat{n} \times \vec{E}) = \omega(\mu \vec{H}).$$

$$\vec{H} = \frac{K}{\omega \mu}(\hat{n} \times \vec{E}) = K = \frac{\omega}{v} \Rightarrow \vec{H} = \frac{1}{\mu v}(\hat{n} \times \vec{E}).$$

or $$\frac{|\vec{E}|}{|\vec{H}|} = \mu v \Rightarrow v = \frac{1}{\sqrt{\mu \epsilon}} \Rightarrow \frac{|\vec{E}|}{|\vec{H}|} = \mu \times \frac{1}{\sqrt{\mu \epsilon}}$$

$$\Rightarrow \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu}{\epsilon}}$$ $\Rightarrow$ The ratio of $\frac{|\vec{E}|}{|\vec{H}|}$ has units Equal to $\frac{\text{Volt}}{\text{amp}}$ = ohm.

$\frac{|\vec{E}|}{|\vec{H}|}$ has same units of those of impedance and it is called as **characteristic or intrinsic impedence of dielectric medium. It is denoted by Z.** It is denoted by Z.

$$Z = \frac{|\vec{E}|}{|\vec{H}|} = \left(\sqrt{\frac{\mu}{\epsilon}}\right).$$

Since, $\mu = \mu_r \mu_0$ and $\epsilon = \epsilon_r \epsilon_0$.

$$Z = \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}} = \sqrt{\frac{\mu_r}{\epsilon_r} \times \frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{\mu_r}{\epsilon_r}} \times Z_0.$$

Where $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ is the impedence of free space.



This is the relationship betn. impedence of dielectric ~~medicine~~ medium and impedence of free space.

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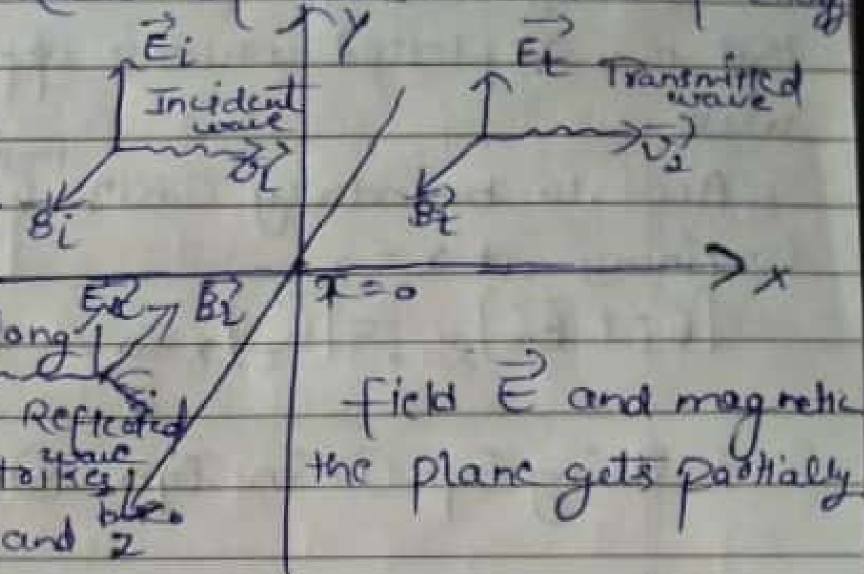
Physics:



# Reflection and Transmission of E.M waves at Normal Incidence at a plane Boundary of two Dielectrics of Law of conservation of Energy




Consider an interface at $x=0$ in two Dielectric media I and II. Let a plane E.m travelling along +ve x-axis with Electric field $\vec{E}$ and magnetic field $\vec{B}$ along z-axis strikes the plane gets partially reflected, transmitted and

Let $\epsilon_1, \epsilon_2$ and $\mu_1, \mu_2$ are the electrical permittivity and electrical ~~permittivity~~ permeabilities of two dielectrics. $v_1, v_2$ are the velocities of e.m in med 1 and 2. $k_1$ and $k_2$ are the propagation vectors in these two media.

The Electric field associated with incident, reflected and transmitted are:

$$\left.\begin{aligned} \vec{E_i} &= E_{0i}\, e^{i(k_1 x - \omega t)}\hat{j} \\ \vec{E_r} &= E_{0r}\, e^{i(-k_1 x - \omega t)}\hat{j} \\ \vec{E_t} &= E_{0t}\, e^{i(k_2 x - \omega t)}\hat{j} \end{aligned}\right\} \text{———} \quad (1).$$

where, $\omega$ is same in both dielectric media 1, 2 and propagation const. are:

$$k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi v}{v_1} = \frac{\omega}{v_1} \text{———} \quad (2).$$

$$k_2 = \frac{\omega}{v_2} \text{———} \quad (3),$$

$\vec{B} = \frac{\vec{E}}{v}$, The magnetic field corresponding to the incident, reflected and transmitted waves are:

$$\left.\begin{aligned}\vec{B}_i &= \frac{1}{v_1} E_{oi}\, e^{i(k_1 x - \omega t)}\hat{k} \\ \vec{B}_r &= -\frac{1}{v_1} E_{or}\, e^{i(-k_1 x - \omega t)}\hat{k} \\ \vec{B}_t &= \frac{1}{v_2} E_{ot}\, e^{i(k_2 x - \omega t)}\hat{k}\end{aligned}\right\} \quad \text{--- (4)}$$

The minus sign shows that the reflected wave is opposite to the incident wave.

Acc. to boundary Condⁿ tang. Components of $\vec{E}$ must be Continuous at $x=0$.

$$[\vec{E}_i + \vec{E}_r]_{x=0} = [\vec{E}_t]_{x=0} \quad \text{--- (5)}$$

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Using Eqn.(i) in Eqn. (5), we get.

$$E_{oi} + E_{or} = E_{ot} \quad \text{--- (6)}$$

Illy $\vec{H}$ must also be Conti. at $x=0$.,

$$[\vec{H}_i + \vec{H}_r]_{x=0} = [\vec{H}_t]_{x=0}, \quad \vec{B} = \mu \vec{H}_{(m.F.I)}$$

mkgn. Induction

$$\text{Now, } \frac{1}{\mu_1}[\vec{B}_i + \vec{B}_r]_{x=0} = \frac{1}{\mu_2}[B_t]_{x=0} \quad \text{--- (7)}$$

Using Eqn. (4) in Eqn. (7) we get,

$$\frac{1}{\mu_1}\left[\frac{E_{oi}}{v_1} - \frac{E_{or}}{v_1}\right] = \frac{1}{\mu_2}\frac{E_{ot}}{v_2}$$

$$E_{oi} - E_{or} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{ot},$$ For Non-magnetic media $\mu_1 = \mu_2 = \mu_0$.

$\frac{\mu_0}{\mu_0}$

$$E_{oi} - E_{or} = \frac{v_1}{v_2} E_{ot} \quad \text{--- (8)}$$

$E_{oi} - E_{or}$

$E_{oi} + E_{or}$

$2E_{oi}$

$= E_{ot}$

Adding Eqn. (6) and (8), we get.

$$2E_{oi} = \left[1+\frac{v_1}{v_2}\right]E_{ot} = \left[\frac{v_1+v_2}{v_2}\right]E_{ot}$$

$$\therefore E_{ot} = \left(\frac{2v_2}{v_1+v_2}\right)E_{oi} \quad \text{--- (9)}$$

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Using Eqn. (9) in Eqn. (6) we get

$$E_{oi}+E_{or} = \left[\frac{2v_2}{v_1+v_2}\right]E_{oi}$$

$$E_{or} = \left[\frac{2v_2}{v_1+v_2}\right]E_{oi} - E_{oi} \Rightarrow E_{or} = \left[\frac{2v_2}{v_1+v_2} - 1\right]E_{oi}$$

$$= \left[\frac{v_2-v_1}{v_1+v_2}\right]E_{oi} \Rightarrow E_{or} = \left[\frac{v_2-v_1}{v_1+v_2}\right]E_{oi} \quad \text{--- (10)}$$

If $v_2 > v_1$, the reflected wave is inphase with incident wave i.e $v_2 < v_1$, the reflected wave is out phase with the incident wave.

$$n = \frac{c}{v} \quad \text{or} \quad v = \frac{c}{n} \quad \text{--- (11)}$$

Eqn. (9) and (10) can be written as:—

$$E_{ot} = \frac{2\,c/n_2}{\left(c/n_1 + c/n_2\right)}E_{oi} = \left[\frac{2n_1}{n_1+n_2}\right]E_{oi} \quad \text{--- (12)}$$

$$E_{or} = \frac{\left[c/n_2 - c/n_1\right]}{\left[c/n_1 + c/n_2\right]}E_{oi} = \left[\frac{n_1-n_2}{n_1+n_2}\right]E_{oi} \quad \text{--- (13)}$$

Intensity of em wave is given by: $I = \frac{1}{2}\epsilon v E_0^2$

Intensity of incident, reflected and transmitted wave :-

$$\Rightarrow I_i = \frac{1}{2}\epsilon_1 v_1 E_{oi}^2$$

$I_r = \frac{1}{2}\epsilon_1 v_1 E_{0r}^2$

$I_t = \frac{1}{2}\epsilon_2 v_2 E_{0t}^2$



(i) Reflection Co-efficient (R): It states that the ratio of intensity of ~~(incident)~~ Reflected wave to the intensity of incident wave is Called Reflection Co-efficient.

$$R = \frac{I_r}{I_i} = \left[\frac{E_{0r}}{E_{0i}}\right]^2$$



Using Eqn. (13) we get., $R = \left[\frac{n_1 - n_2}{n_1 + n_2}\right]^2$ — (14).

(ii) Transmission-Co-efficient (T): It states that the ratio of intensity of transmitted wave to the intensity of incident wave.

$$T = \frac{I_t}{I_i} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left[\frac{E_{0t}}{E_{0i}}\right]^2$$

Using Eqn. (12) we get

$$T = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\left(\frac{2n_1}{n_1 + n_2}\right)^2, \quad n \propto \sqrt{\epsilon} \text{ and } n \propto \frac{1}{v}$$

$$\frac{\epsilon_2 v_2}{\epsilon_1 v_1} = \frac{n_2^2 \cancel{n_1}}{n_1^2 \cancel{n_2}} = \frac{n_2}{n_1} \Rightarrow T = \frac{n_2}{n_1}\left[\frac{2n_1}{n_1 + n_2}\right]^2$$

$$\boxed{T = \frac{4n_1 n_2}{(n_1 + n_2)^2}} \quad \text{— (15)}$$

Adding (14) and (15), we get.

$$R + T = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} + \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

$$R + T = \frac{n_1^2 + n_2^2 - 2n_1 n_2 + 4n_1 n_2}{(n_1 + n_2)^2} = \frac{n_1^2 + n_2^2 + 2n_1 n_2}{(n_1 + n_2)^2}$$

$$R + T = \frac{\cancel{(n_1 + n_2)^2}}{\cancel{(n_1 + n_2)^2}} = 1. \Rightarrow R + T = 1$$

This is the law of Conservation of Energy.

## **Reflection and Transmission of Electromagnetic waves at Oblique incidence at a plane boundary of Dielectric

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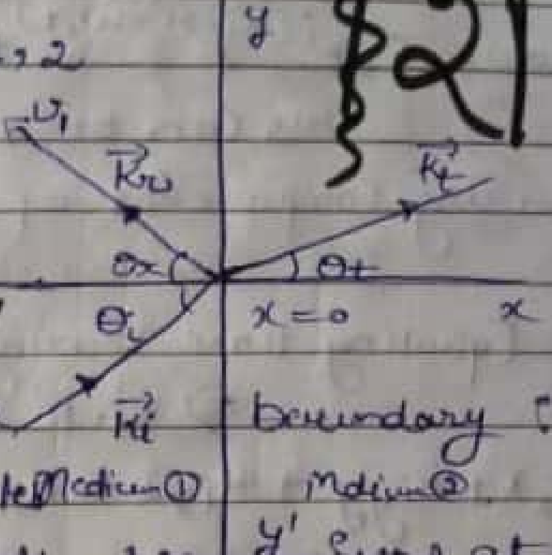
Consider two dielectric media 1, 2 seperated by $yy'$ at $x=0$. Let a plane Em waves with wave vector $\vec{K_i}$ and angular frequency $\omega_i$ be incident at an angle $\theta_i$ on a plane boundary :




It is partially reflected at an angle $\theta_r$ and partially transmitted at an angle $\theta_t$. $n_1$ and $n_2$ are the refractive indices. $\epsilon_1$ and $\epsilon_2$ are the Electrical permeabilities of media 1 and 2.

$\vec{K_r}$ and $\vec{K_t}$ be the wave vector of reflected and transmitted wave.

$\vec{\omega_r}$ and $\vec{\omega_t}$ are frequencies of reflected and transmitted wave.

For incident wave,

$$\vec{E_i} = E_{0i}e^{i(\vec{K_i}\cdot\vec{r} - \omega_i t)}$$

$$\vec{B} = \frac{\vec{K_i} \times \vec{E_i}}{\omega_i}$$

For Reflected wave

$$\vec{E_r} = E_{0r}e^{i(\vec{K_r}\cdot\vec{r} - \omega_r t)}$$

$$\vec{B_r} = \frac{\vec{K_r} \times \vec{E_r}}{\omega_r}.$$

For transmitted wave:

$$\vec{E_t} = E_{0t} e^{i(\vec{k_t}\cdot\vec{x} - \omega_t t)}$$

$$\vec{B_t} = \frac{\vec{k_t} \times \vec{E_t}}{\omega_t}$$ , Acc. to boundary cond$^n$,

$\vec{E}$ is continous at $x = 0$

Now, $E_{0i} e^{i(\vec{k_i}\cdot\vec{x} - \omega_i t)} + E_{0r} e^{i(\vec{k_r}\cdot\vec{x} - \omega_r t)} = E_{0t} e^{i(\vec{k_t}\cdot\vec{x} - \omega_t t)}$

This can be satisfied its time and space varing components of phase are Equal.

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Equating time varing components,

$$\Rightarrow \vec{k_i}\cdot\vec{x} = \vec{k_r}\cdot\vec{x} = \vec{k_t}\cdot\vec{x}.$$

$$k_{ix}x + k_{iy}y + k_{iz}z = k_{rx}x + k_{ry}y + k_{rz}z = k_{tx}x + k_{ty}y + k_{tz}z.$$

At interface $x = 0$.

$$k_{iy}y + k_{iz}z = k_{ry}y + k_{rz}z = k_{ty}y + k_{tz}z.$$

$y$ and $z$ are independent

$$k_{iy} = k_{ry} = k_{ty}.$$

$$k_{iz} = k_{rz} = k_{tz}.$$

Let $\vec{k_i}$ lies in $xy$ plane.

But $k_{iz} = 0$.

Then, $k_{rz} = k_{tz} = 0$.

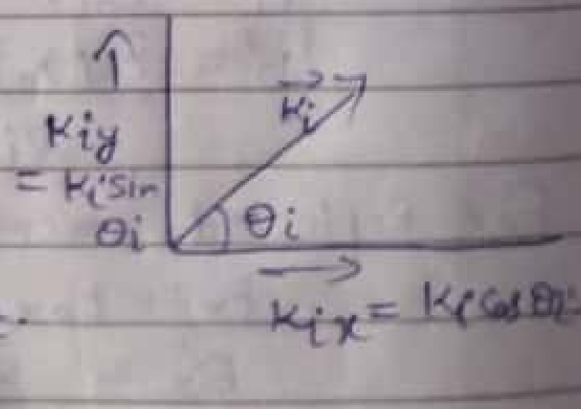
$\vec{k_r}$ and $\vec{k_t}$ also lie in $x$-$y$ plane.

Normal $\hat{n}$ is along $x$-axis,

$$k_{iy} = k_i \sin\theta_i$$

$$k_{ry} = k_r \sin\theta_r$$

$$k_{ty} = k_t \sin\theta_t.$$




From Eqn (1)

$$k_i \sin\theta_i = k_r \sin\theta_r = k_t \sin\theta_t.$$

$\Rightarrow k_i = k_r$ , $k_i \sin\theta_i = k_t \sin\theta_t$

$\sin\theta_i = \sin\theta_r$ $\quad k_i = \frac{\omega}{v_1}$ , $k_t = \frac{\omega}{v_2}$

$\theta_i = \theta_r$.

$$\frac{\omega}{v_1}\sin\theta_i = \frac{\omega}{v_2}\sin\theta_t. \Rightarrow \frac{1}{v_1}\sin\theta_i = \frac{1}{v_2}\sin\theta_t$$

$$\frac{c}{v_1}\sin\theta_i = \frac{c}{v_2}\sin\theta_t$$

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$n_1 = \frac{c}{v_1}$ ref. index of medium (1).

$n_2 = \frac{c}{v_2}$, ,, ,, ,, ,, (2).

$$n_1 \sin\theta_i = n_2 \sin\theta_t.$$

$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1}$ , This is known as Snell's law.

* Reflection and Transmission Co-efficients:

Applying boundary Condⁿ :-

(i) $[\vec{E_i} + \vec{E_r}]_{x=0} = [\vec{E_t}]_{x=0}$.

$$E_{0i}\cos\theta_i + E_{0r}\cos\theta_r = E_{0t}\cos\theta_t \quad \text{--- (3)}.$$

(ii) $\frac{1}{\mu_1}[\vec{B_i} + \vec{B_r}]_{x=0} = \frac{1}{\mu_2}[\vec{B_t}]_{x=0}$

$$\frac{1}{\mu_1 v_1}[E_{0i} - E_{0r}] = \frac{1}{\mu_1 v_1} E_{0t}$$

$\frac{\mu_1 v_1}{\mu_2 v_2} = A$ Assume.

$E_{0i} - E_{0r} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0t}$ , * $v = \frac{c}{n}$ and $v_2 = \frac{c}{n}$

Now, $E_{0i} - E_{0r} = A E_{0t}$ --- (4)

$$A = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1} \quad \text{--- (5)}.$$

$\theta_i = \theta_r$

Now, $E_{oi} + E_{or} = \left[\frac{\cos\theta_t}{\cos\theta_i}\right] E_{ot}$

or



$$E_{oi} + E_{or} = B\,E_{ot} \quad \text{--- (6)}$$

$$B = \frac{\cos\theta_t}{\cos\theta_i} \quad \text{--- (7)}$$

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Adding Eqn. (4) and (6) we get.

$$2E_{oi} = (A+B)E_{ot}$$

$$\boxed{E_{ot} = \left[\frac{2}{A+B}\right] E_{oi}} \quad \text{--- (8)}$$

From Eqn. (6), we get.

$$E_{or} = B\,E_{ot} - E_{oi} = \left[\frac{2B}{A+B}\right] E_{oi} - E_{oi}$$

$$= \left[\frac{B-A}{A+B}\right] E_{oi} \quad \text{--- (9)}$$ , Eqn. (8) and (9) are known as Fresnel's Eqn.

From Eqn. (7). $B = \frac{\sqrt{1-\sin^2\theta_t}}{\cos\theta_i}$

Acc. to Snell's law

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1} \text{ or } \sin\theta_t = \left[\frac{n_1}{n_2}\right]\sin\theta_i$$

$$B = \frac{\sqrt{1-\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i}}{\cos\theta_i} \quad \text{--- (10)}$$

(i) If $\theta_i = 0$, $B = 1$ Eqn. (8) and (9),

$$E_{ot} = \left[\frac{2}{1+A}\right] E_{oi} = \frac{2E_{oi}}{\left[1+\frac{\mu_1 n_2}{\mu_2 n_1}\right]} \quad \text{--- (11)}$$

Trans.

$$E_{or} = \left[\frac{1-A}{1+A}\right] E_{oi} = \frac{\left[1-\frac{\mu_1 n_2}{\mu_2 n_1}\right]}{\left[1+\frac{\mu_1 n_2}{\mu_2 \mu_1}\right]} E_{oi}$$

$\left(\frac{\mu_2 n_1 - \mu_1 n_2}{\mu_2 n_1 + \mu_1 n_2}\right) E_{0i}$ —— (12).

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~~If $\theta_i = 90°$ and $\theta_i = \theta_B$ Brewster's angle~~ angles of incidence, reflected and transmitted disappears i.e $E_{0R} = 0$

From Eqn. (9), $A = B$.

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$$B = \frac{\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_B}}{\cos\theta_B}, \quad 1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_B = B \cos^2\theta_B$$

$$\sin^2\theta_B = \frac{1 - B^2\cos^2\theta_B}{\left(\frac{n_1}{n_2}\right)^2} = \frac{1 - B^2(1-\sin^2\theta_B)}{\left(\frac{n_1}{n_2}\right)^2}$$

$$\Rightarrow \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_B - B^2 \sin^2\theta_B = 1 - B^2.$$

$$* \sin^2\theta_B = \frac{1 - B^2}{\left(\frac{n_1}{n_2}\right)^2 - A^2} \quad \{B = A\}.$$

$$\Rightarrow \frac{1 - \left(\frac{\mu_1 n_2}{\mu_2 n_1}\right)^2}{\left(\frac{n_1}{n_2}\right)^2 - \left(\frac{\mu_1 n_2}{\mu_2 n_1}\right)^2} \quad \text{if } \mu_1 = \mu_2.$$

$$\sin^2\theta_B = \frac{1 - \left(\frac{n_2}{n_1}\right)^2}{\left(\frac{n_1}{n_2}\right)^2 - \left(\frac{n_2}{n_1}\right)^2} = \frac{n_1^2 - n_2^2 / n_1^2}{\left(\frac{n_1^2}{n_2^2} - \frac{n_2^2}{n_1^2}\right)} = \frac{(n_1^2 - n_2^2)/n_1^2}{n_1^4 - n_2^4 / n_1^2 n_2^2}$$

$$= \frac{n_2^2 \cancel{(n_1^2 - n_2^2)}}{\cancel{(n_1^2 - n_2^2)}(n_1^2 + n_2^2)} = \frac{n_2^2}{(n_1^2 + n_2^2)}.$$

—— (13).

$$\Rightarrow \sin\theta_B = \frac{n_2}{\sqrt{n_1^2 + n_2^2}}$$

$$\Rightarrow \cos\theta_B = \sqrt{1 - \sin^2\theta_B} = \sqrt{1 - \frac{n_2}{n_1^2 + n_1^2}}$$

$$\cos\theta_B = \frac{n_1}{\sqrt{n_1^2 + n_2^2}}$$ —— (14).

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$$\tan\theta_B = \frac{\sin\theta_B}{\cos\theta_B} = \left[\frac{n_2}{n_1}\right] \text{ — (15).}$$

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Intensity of wave making at an angle $\theta$ with the normal to the surface is :—

$$I = \frac{1}{2}\epsilon v E_0^2 \cos\theta.$$

i.e Intensities of incident wave, reflected wave and transmitted wave :—

$$I_i = \frac{1}{2}\epsilon_1 v_1 E_{0i}^2 \cos\theta_i.$$

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$$I_r = \frac{1}{2}\epsilon_1 v_1 E_{0r}^2 \cos\theta_r.$$

$$I_t = \frac{1}{2}\epsilon_2 v_2 E_{0t}^2 \cos\theta_t.$$

Reflection Co-effi. for e.m wave is :

$$R = \frac{I_r}{I_i} = \left[\frac{E_{0r}}{E_{0i}}\right]^2 \frac{\cos\theta_r}{\cos\theta_i} = \left[\frac{E_{0r}}{E_{0i}}\right]^2, \quad \{\theta_i = \theta_r\}$$

Using Eqn. (9). $R = \left(\frac{B-A}{A+B}\right)^2$ — (17).

The transmission Co-effi. of e.m wave

$$T = \frac{I_t}{I_i} = \left[\frac{\epsilon_2 v_2}{\epsilon_1 v_1}\right]\left[\frac{E_{0t}}{E_{0i}}\right]^2\left[\frac{\cos\theta_t}{\cos\theta_i}\right] \text{ — (18).}$$

$$T = \frac{1}{\sqrt{\mu\epsilon}}, \quad {}^{*}\epsilon = \frac{1}{\sqrt{\mu v^2}}$$

$$\frac{\epsilon_2 v_2}{\epsilon_1 v_1} = \frac{\cancel{v_2}}{\cancel{v_1}} \times \frac{\mu_1 v_1^{\cancel{2}}}{\mu_2 v_2^{\cancel{2}}} = \frac{\mu_1 v_1}{\mu_2 v_2} = A \text{ — (19).}$$

Using Eqn. (7), (8) and (19) in Eqn. (18).

$$T = AB\left[\frac{2}{A+B}\right]^2 \text{ — (20).}$$

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Adding Eqn. (17) and (20),

$$R+T = \left[\frac{B-A}{A+B}\right]^2 + \frac{4AB}{(A+B)^2} = 1. \quad (21)$$

$$\frac{B^2+A^2-2BA+4AB}{(A+B)^2} = \frac{B^2+A^2+2AB}{A^2+B^2+2AB} = 1.$$

$$\boxed{R+T=1}.$$

Which is law of Conservation of Energy. Ans

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